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FERMATEAN FUZZY HYPERGRAPH

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Abstract
Fermatean Fuzzy Sets(FFSs) is an extension of the orthopair fuzzy sets which can be able to carried out uncertain evaluations more actively in decision-making environment. In this paper we proposes the new concept of Fermatean Fuzzy Hyper- graphs(FFHGs).The basic definitions of hypergraphs under Fermatean fuzzy environment are initiated and examined.The size and order of Fermatean fuzzy hypergraph are defined. Also,the regular and dual fermatean fuzzy hypergraph are investigated.Some of its properties are analysed by a suitable example.
Keywords: Fermatean fuzzy graph,Fermatean fuzzy hypergraph,Regular FFHG,Dual FFHG.
抽象的
Fermatean Fuzzy Sets（FFSs）**是正交**对模糊集的扩展，能够在决策环境中更主动地进行不确定性评价。 **在本文中，我们**提出了费马模糊超图（FFHGs）**的新概念。提出并**检验了费马模糊环境下超图的基本定义。定义了费马模糊超图的大小和阶。 还研究了正则和对偶费马模糊超图，并通过一个合适的例子分析了它的一些性质。

关键词：Fermatean **模糊**图，Fermatean **模糊超**图，Regular FFHG，Dual FFHG。

1 Introduction

Fermatean fuzzy environment is the modern tool for handling uncer- tainty in many decisions making problems. Rosenfeld [1] considered fuzzy relations on fuzzy sets and later developed the theory of fuzzy graph in 1975. Atanassov [2] elongated the fuzzy sets to intuitionistic fuzzy sets (IFSs) which incorporated the degree of hesitation called hes- itation margin $\pi = 1 - \mu - \nu$ with membership values μ and non-membership values ν. Shannon and Atanassov [3] proposed the concept of intuitionistic fuzzy graphs by consider- ing intuitionistic fuzzy relations on IFSs.

Orthopair fuzzy sets are defined by denoting fuzzy membership grades

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not only with a single value but described by pairs of mutually orthogonal fuzzy sets which accept the integration of uncertainty. Atanassov [3] introduced the intuitionistic fuzzy sets which has the sum of the membership grade and the non-membership grade must be equal to or less than one. But there may be a problem that these condition of an IFS is not satisfied in some situation . To overcome this situation Yager[4] developed a new type of fuzzy sets called the Pythagorean fuzzy sets which satisfies the constraint that the sum of the squares of the membership grades nd the non-membership grade is equal to or less than one. But they still have obvious shortcomings in which PFS and IFS are failed to overcome a situation in which the sum of the membership grade and the non-membership grade is greater than 1 and sum of the squares is still greater than 1. Inorder to relax this situation, Senapati and Yager [7] developed a modern idea called Fermatean fuzzy set that has the constraints as the sum of the third power of the membership grades and the non-membership grades is less than 1.FFSs contains a stronger capacity to support uncertain information by emerging the spatial scope of membership and non-membership grade. Senapati and Yager [7] define basic operations over Fermatean Fuzzy Sets in 2019. Yager [4] proposed a q-rung orthopair fuzzy sets, as a new conception of orthopair fuzzy sets in 2017.

Kaufman [8] define the basic concept of fuzzy hypergraphs which was generlalized and modified by Lee-Kwang and Lee [9]. In this study, we proposed the new idea of Fermatean Fuzzy Hypergraph which is the generalization of the intuitionistic fuzzy hypergraphs.

2. Preliminaries

study is given.

Definition 2.1[11]

In this section ,the basic notions and terminologies associated to the

The pair A H = (V, E) is called a Hypergraph H if

- V = {v1, v2, · · · , vn}, be a finite set
- E = {E1, E2, · · · , En}, be a set of non-empty subsets of V
- Ej , j = 1, 2, · · · , m and ∪Ej = V .

The elements {v1, v2, · · · , vn} are vertices of hypergraph H and the set {E1, E2, · · · , En} are hyperedges of the hypergraph H.

Definition 2.2[11]

In a hypergraph two or more vertices v1, v2, · · · , vn are called adjacent

if there is an edge Ej between those two vertices.

In a hypergraph two edges Ei and Ej,

i j are called adjacent if the intersection between those two edges is not empty. That is

Ei ∩ Ej /=, i /= j.

Definition 2.3[13]

If every vertex of a hypergraph has degree k, then the hypergraph is

called k-regular hypergraph . A hypergraph H is called k-uniform if the cardinality k is same for all the edges.

Definition 2.4[11]

The dual hypergraph H∗ is defined to be a hypergraph whose vertices and edges are interchanged, so that the vertices are given by ei and whose edges are given by Vi where Vi = {ei/vi ∈ Ei} .

Definition 2.5[7]

A Fermatean fuzzy set(FFS) F on a universe of discourse X is a struc- ture having the form as

F = {⟨x, muF (x), νF (x)⟩ /x ∈ X}

where µF (x) : X → [0, 1] and νF (x) : X → [0, 1] indicate the membership grades and the non-

membership grades of every element x ∈ X along with the constraints 0 ≤ μ3 (x)+ν3 (x) ≤ 1
√
satisfies for all x ∈ X. π (x) = 3 1 − μ3 (x) − ν3 (x) provides the degree of uncertainty of x
F F F
to F .
Definition 2.6[7]
Supp(F) is the Support of an FFS F which is defined as Supp(F) = {vi : μF (vi) > 0 and νF (vi) > 0}.Supp(F) is a normal crisp value . Definition 2.7[7]
The (α, β)-cut of a Fermatean fuzzy set F , indicated by F(α,β) given
by
3. Main results
F(α,β) = {vi : μF (vi) ≥ α and νF (vi) ≤ β}.
Fermatean Fuzzy Hypergraph Definition 3.1
A Fermatean Fuzzy Graph(FFG) is of the form GF =(XF , EF), where
XF = {x1, x2, · · · , xn} such that μ1 : XF → [0, 1] and ν1 : XF → [0, 1] represent the membership and non-membership grades of the element xi ∈ XF respectively and 0 ≤ μ3(x)+ν3(x) ≤
1 1
1 for every x ∈ XF and EF ⊂ XF ×XF where μ2 : XF ×XF → [0, 1] and ν2 : XF ×XF → [0, 1] denote the membership and non-membership grades of the element ei∈ EF such that
μ2(xi, xj) ≤ min(μ1(xi), μ1(xj))
ν2(xi, xj) > max(ν1(xi), ν1(xj))
0 ≤ μ3(xi, xj) + ν3(xi, xj) ≤ 1 for every (xi, xj) ∈ EF .
2 2
Definition 3.2
A Fermatean Fuzzy Hypergraph(FFHG) is represented as a structure of a pair of the form HF = (XF , EF) which is given as

- XF = {a1, a2, · · · , an}, which is a finite set of vertices
- EF = {E1, E2, · · · , Em}, which is a non-empty Fermatean fuzzy subsets of X
- EF j = {(ai, μj(ai), νj(ai)/(μj(ai), νj(ai)) ≥ 0) and 0 ≤ μ3(ai, aj) + ν3(ai, aj) ≤ 1,
j j
j = 1, 2, · · · , m}
- EF j ∅, j = 1, 2, · · · , m. where (μj(ai), νj(ai)) denotes the membership and the non-membership grades of the element ai ∈ XF respectively of the vertex ai to edge EF j.

Example 1
Consider a FFHG HF = (XF , EF) where XF = {a1, a2, a3, a4} and
EF = {EF 1, EF 2}. Here EF 1 = {a1, a2, a3} and EF 2 = {a3, a4}. The FFHG is shown on Figure 1.

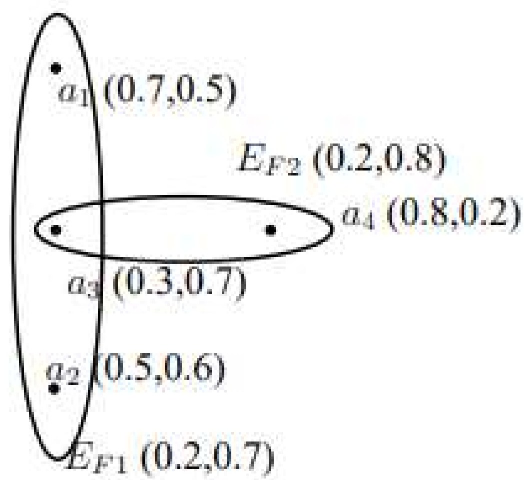

Figure - 1 FFHG

Definition 3.3
A FFHG HF = (XF , EF) is said to be a μ-srtong FFHG if
μ2ij = min(μ1i, μ1j) for all (ai, aj) ∈ EF

Example 2

Consider a FFHG, HF = (XF , EF) with XF = {a1, a2, a3, a4} and E = {EF 1, EF 2}. Here EF 1 = {a1, a2, a3} and EF 2 = {a2, a4}. The μ-strong FFHG is presented in Figure-2.

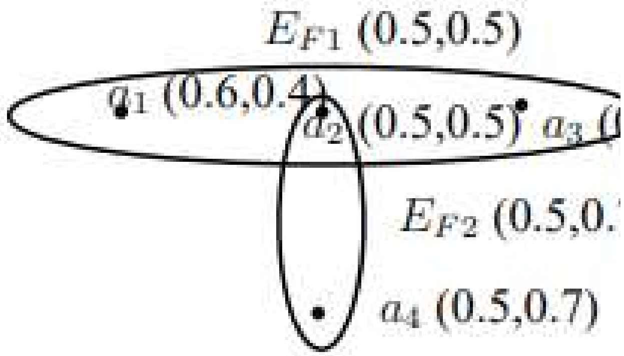

Figure - 2 μ-Strong FFHG

Definition 3.4
A FFHG, HF = (XF , EF) is said to be a ν-srtong FFHG if

Example 3
ν2ij = max(ν1i, ν1j) for all (ai, aj) ∈ EF Consider a FFHG, HF = (XF , EF) with XF = {a1, a2, a3, a4} and E = {EF 1, EF 2}. Here EF 1 = {a1, a2, a3} and EF 2 = {a3, a4, a5}. The ν-strong FFHG is displayed in Figure-3.

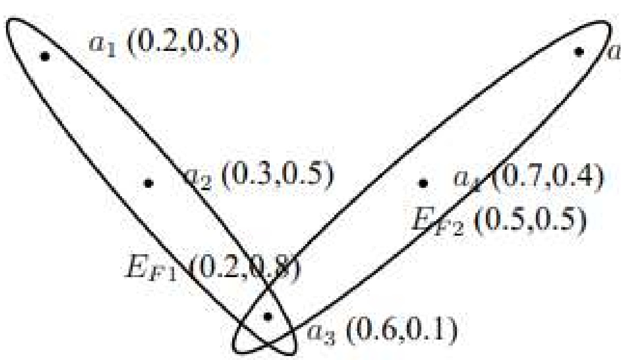

Figure - 3 ν-strong FFHG

Definition 3.5
Example 4
A FFHG HF = (XF , EF) is said to be a Strong FFHG if μ2ij = min(μ1i, μ1j) and ν2ij = min(ν1i, ν1j) for all (ai, aj) ∈ EFConsider a FFHG, HF = (XF , EF) with vertex XF = {a1, a2, a3} and EF = {E1, E2}. Here EF 1 = {a1, a2, a3} and EE2 = {a2, a3}. The Strong FFHG is represented in Figure-4.

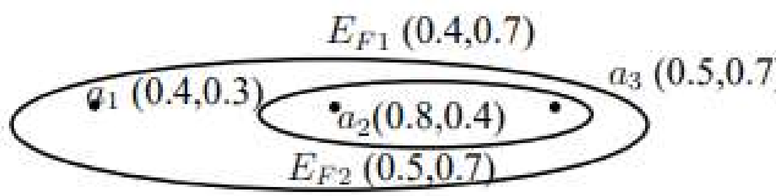

Figure- 4

Definition 3.6
The set consisting of adjacent vertices of ai with the elimination of the
vertex ai is called the open neighbourhood of the vertex ai in the FFHG and is represented by N (ai) .
Example 5
In Example 3, the vertex a1 has the open neighbourhood a2 and the vertex a3 has a2 and a4.
Definition 3.7
The set containing the adjacent vertices of ai with the inclusion of the vertex ai also is called the closed neighbourhood of the vertex ai in the FFHG and is denoted by N [ai].
Example 6
In Example 3, the vertex a1 has the closed neighbourhood as a1 and the vertex a2 has a3, a2 and a4.
Definition 3.8

In a FFHG HF , the degree of open neighbourhood of a vertex ai, rep- resented by Δ(ai) which is given as
Δ(ai) = (Δμ(ai), Δν(ai)) where
Δμ(a) = Σ
μEF (ai)
Example 7
Definition 3.9
ai∈ΣN (ai)
Δν(ai) = νEF (ai)
ai∈N (ai)
In Example 3, Δ(a5) is (1,0.9) .
In a FFHG FH , the degree of closed neighbourhood of the vertex ai,
denoted as δ[ai] which is given by
δ[ai] = (δμ[ai], δν[ai]) where
δμ[a] = δμ(a) + Σ
μEF (ai)
δν[ai] = δν(ai) + ai∈ΣN (ai) νEF (ai)
ai∈N (ai)

Example 8
Definition 3.10
In Example 3, δ[a3] is (1.6,1) .
The n-regular FFHG is defined as in which the open neighbourhood degree of all the vertices in XF has n.
Definition 3.11
The m-totally regular FFHG is defined as in which the closed neigh- bourhood degree of all the vertices in XF has m.
Definition 3.12
Let HF = (XF , EF) be a regular FFHG. Then the order of a regular
Σ
FFHG HF is O(HF) = (
ai∈XF
Σ
μEF (ai),
ai∈XF
νEF (ai)) for every ai ∈ XF . The size of regular
Σn
FFHG is S(HF) =
i
Example 9
S(EF i) where S(EF i) = (
Σ
ai∈Ei
μEF (ai),
Σ
ai∈EF i
νEF (ai)).

Consider a n-regular and m-totally regular FFHG , HF = (XF , EF).
Define XF = {a1, a2, a3} and EF = (EF 1, EF 2, EF 3) where
EF 1 = {(a1, 0.5, 0.4)(a2, 0.5, 0.4)}
EF 2 = {(a2, 0.5, 0.4)(a3, 0.5, 0.4)}
EF 3 = {(a3, 0.5, 0.4)(a4, 0.5, 0.4)}
Then the order of FFHG, O(HF) = (1.5, 1.2) and the size of FFHG, S(HF) = (3.0, 2.4)
Proposition 3.1 : The order of n-regular FFHG HF is nk , |XF | = k.
Proposition 3.2 : The FFHG HF has O(HF) = k(m − n), where |XF | = k when it is both
n-regular and m-totally regular.
Proposition 3.3 : A m-totally regular FFHG , HF has 2S(HF) − O(HF) = mk, |XF | = k. Theorem 3.1 : Consider a FFHG HF = (XF , EF) of a hypergraph H. Then μEF : X → [0, 1], νEF : X → [0, 1] is a constant function and the resulting statements are identical.
(i) HF is a regular FFHG
(ii) HF is totally regular FFHG
Proof:
Assume that (μEF , νEF) is a constant function. Let μEF (a) = c1 and
νEF (a) = c2 for all a ∈ Ei and (c1, c2) ∈ [0, 1].

Assume that HF is n-regular FFHG. Then $\Delta\mu(a) = n1$ and $\Delta\nu(a) = n2$ for all $a \in$ EF i. So

$\delta\mu[a] = \mu(a) + \mu EF(a)$ for all $a \in$ EF i

$\delta\nu[a] = \Delta\nu(a) + \nu EF(a)$ for all $a \in$ EF i, $i = 1, 2, \cdots, n$.

Thus $\delta\mu[a] = n1 + c1$, $\delta\nu[a] = n2 + c2$ for all $a \in$ EF i. Hence, HF is totally regular FFHG. Thus (i)⇒(ii).

Assume that HF is a m-totally regular FFHG. Then $\delta\mu[a] = k1$ and

$\delta\nu[a] = k2$ for all $a \in$ EF i, $i = 1, 2, \cdots, n$

$\Rightarrow \delta\mu[a] + \mu EF(a) = k1$ for all $a \in$ EF i and

$\delta\nu[a] + \nu EF(a) = k2$ for all $a \in$ EF i

$\Rightarrow \Delta\mu(a) + c1 = k1$ for all $a \in$ EF i and

$\Delta\nu(a) + c2 = k2$ for all $a \in$ EF i

$\Rightarrow \Delta\mu(a) = k1 - c1$ for all $a \in$ EF i and

$\Delta\nu(a) = k2 - c2$ for all $a \in$ EF i

Thus HF is a regular FFHG. Hence the statements(i) and (ii) are identical. Obviously we can prove the converse part.

Theorem 3.2 : A FFHG, HF has (μEF , νEF) as constant function if it is regular and also totally regular.

Proof:

Let HF be both regular and totally regular FFHG. Then $\Delta\mu(a) = n1$,

$\Delta\nu(a) = n2$ for all $a \in$ EF i and $\delta\mu[a] = k1$, $\delta\nu[a] = k2$ for all $a \in$ EF i.

$\Leftrightarrow \Delta\mu(a) + \mu EF(a) = k1$ for all $a \in$ EF i

$\Leftrightarrow n1 + \mu EF(a) = k1$ for all $a \in$ EF i

$\Leftrightarrow \mu EF(a) = k1 - n1$ for all $a \in$ EF i

$\Leftrightarrow \delta\mu[a] = k2$ for all $a \in$ EF i

$\Leftrightarrow \Delta\nu(a) + \nu E(a) = k2$ for all $a \in$ EF i

$\Leftrightarrow n2 + \nu EF(a) = k2$ for all $a \in$ EF i

$\Leftrightarrow \nu EF(a) = k2 - n2$ for all $a \in$ EF i

Hence (μEF , νEF) is constant function.

Definition 3.13

In a FFHG, the level of adjacency between the two vertices ar and as are represented by Γ(ar, as) which is defined as Γ(ar, as) = maxj(min(μj(ar), μj(as))), minj(max(νj(ar), νj(as))), $j = 1, 2, \cdots, m$.

Definition 3.14

In a FFHG, the level of adjacency between the two edges EF j and EF k, denoted by σ(EF j, EF k) is defined as σ(EF j, F) = maxj(min(μj(ai), μk(ai))), minj(max(νj(ai), νk(ai))).

Example 9

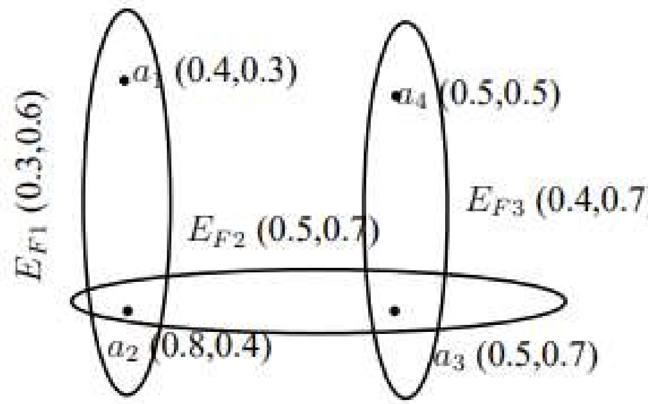

Figure - 5

Here, the level of adjacency between the vertices a1 and a2 is (0.4,0.4) and the level of adja- cency between the two edges EF 1 and EF 2 is (0.5,0.7).

Definition 3.15

The FFHG HF has (α, β)-cut of the form HF (α, β) which is given by a pair HF (α, β) = (XF (α,β), EF (α,β)) where

(i) XF (α,β) = {a1, a2, · · · , an}

(ii) EF (α,β) = {ai : μj(ai) ≥ α and νj(ai) ≤ β, j = 1, 2, · · · , m}.

(iii) EF m+1(α,β) = {ai : μj(ai) < α and νj(ai) > β} for every j.

Definition 3.16

The Strength Ω of an edge EF j is the least membership degree μj(ai) and non-membership degree νj(ai) of vertices in the edge EF j.

Ω(EF j) = (minai μj(ai), maxai νj(ai)) for every μj(ai) > 0, νj(ai) > 0.

Example 10

In Example 9, the strength of edge is Ω(EF 1) = (0.4, 0.4) = Ω(EF 2) = (0.5, 0.7) and Ω(EF 3) = (0.5, 0.7).

Definition 3.17

The Dual Fermatean Fuzzy Hypergraph (DFFHG) of the FFHG, HF =
(XF , EF 1, EF 2, · · · , EF m), X = (a1, a2, · · · , an) is defined as H∗ = (EF , a¯1, a¯2, · · · , a¯n) where

(i) EF = (e1, e2, · · · , em), a finite set called vertices corresponding to EF 1, EF 2, · · · , EF m re- spectively.

(ii) a¯j = {(ej, μj(ej), νj(ej))/μj(ej) = μj(aj), νj(ej) = νj(aj)}.

Example 11

In Example 9, EF = {E1, E2, , E3} and XF = {a1, a2, a3} where

EF 1 = {(a1, 0.4, 0.3)(a2, 0.8, 0.4)}

EF 2 = {(a2, 0.8, 0.4)(a3, 0.5, 0.7)}

EF 1 = {(a3, 0.5, 0.7)(a4, 0.5, 0.5)}

The Dual FFHG HF ∗={E,a¯1,a¯2,a¯3,a¯4} where

The DFFHG is shown below

EF = {e1, e2, e3, e4} a¯1 = {(e1, 0.4, 0.3)}

a¯2 = {(e1, 0.8, 0.4)(e2, 0.8, 0.4)}

a¯3 = {(e2, 0.5, 0.7)(e3, 0.5, 0.7)}

a¯4 = {(e4, 0.5, 0.5)}

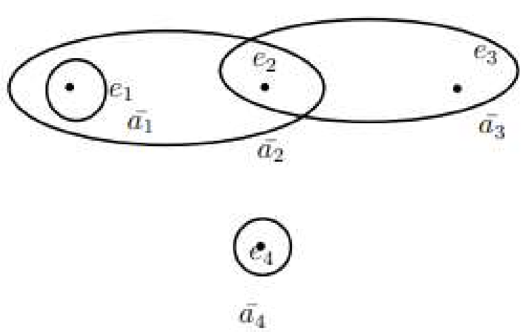

Figure - 6 DFFHG

4 Conclusion

The author put forward to introduce the notion of Fermatean fuzzy
hypergraph(FFHG). The strength Ω of an edge , (α, β) cut and the dual of FFHG is defined with example.Also,regular and m-totally is derived and some of its properties are investigated under FFHG.In future, the FFHG is applied in Network analysis and Clustering problems.

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