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A SINGLE SERVER FINITE CAPACITY QUEUE WITH BALKING AND RETENTION OF IMPATIENT CUSTOMERS

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Abstract

This paper deals with a single server queueing system with finite capacity. We check the performance of the Markovian feedback queue with balking, reneging, and retention of the reneged customer. Customer dissatisfaction as a result of poor service quality is referred to as feedback in queueing literature. Customer retries service after receiving partial or incomplete service in the case of feedback. Balking is a customer's behavior when they come into the queueing system leaves the system without entering; this may rise to potential harm in revenue for the service provider. Further, some customers who arrive at the queueing system join the waiting line and leave due to impatiens, this type of behavior is called reneging. Model's steady-state solution is found. Some numerical and graphical of performance measures are also calculated using the MATLAB and MS-Excel.

Keywords: Feedback, Balking, Reneging, Probability of Customer Retention, Performance Analysis.

1. Introduction

Many researchers have been attracted to performance modeling of Markovian queues with different customer behaviour for the reason that of their extensive variety of applications in practical situations such as inventory systems, hospital emergency rooms dealing with critical patients, computer and communication systems, impatient telephone switchboard customers, and manufacturing and production systems. In queueing literature customer dissatisfaction is called feedback because of unsuitable quality of service.

(Takacs, 1962) studies single server queue with feedback and determine the queue size distribution, the Laplace-Stieltjes transform. (Wortman et al., 1991) M/GI/1 Bernoulli feedback queue is investigated, where the server goes on vacation according to Bernoulli schedules. (Jewkes & Buzacott, 1991) examine queueing system of a K class M/G/1 with feedback. (Van den Berg & Boxma, 1991) feedback mechanism of an M/M/1 queue with a general probability. (Choi et al., 1998, 2000) consider M/M/c retrial queues with feedback and geometric loss and multi-class customers and Bernoulli feedback policy of an M/G/1 queue.

Retrial queue with Bernoulli feedback discusses multi-server feedback retrial queues with balking and control retrial rate, as well as a generalized M/G/1 feedback queue in which customers are either "positive" or "negative" studied by (Kumar et al., 2002, 2006; Kumar & Raja, 2006). (Thangaraj &

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Vanitha, 2009a, 2009b, 2010) In an M/M/1 feedback queue with the possibility of catastrophes at the service counter and a two-phase M/G/1 queueing system with Bernoulli feedback where the server takes multiple vacations, the system size is calculated analytically using continued fractions. (Ke & Chang, 2009) studies balking and Bernoulli feedback with a general retrial queue, where modified vacation policy is operates by the server. (Jeeva & Rathna Kumari, 2012) give a mathematical method to concept the membership function with a non-linear programming of the M/G/1 system, feedback, and bulk arrival queues with server vacation facility, in which departure probability, service time, arrival rate, vacation time, and batch size are all fuzzy numbers.

The transient and reliability analysis of the M/M/1 feedback queue is investigated when it is subjected to catastrophes, server failures, and repairs by (Chandrasekaran & Saravanarajan, 2012). (K. Dwivedi & Gupta, 2014) described performance analysis of Queue Scheduling of Multilevel Feedback based on simulator. (Akhdar, 2020) presents a transient solution that is obtained analytically through using Laplace transforms on processing the probability-generating function independent upon the theory of complex analysis to Rauch regarding size of the system with possibility of catastrophes at services failures and repairs of an M/M/2 feedback queue. (Ayyappan & Sathiya, 2013) consider Server Vacations and Three-Stage Heterogeneous Service having Restricted Admissibility of $M^{[x]}/G/1$ Feedback Queue. (Azhagappan & Deepa, 2019) analyses the control of admission during vacation, interrupted closedown time, feedback, with single vacation transient behaviour of an M/M/1queueing model.

A queueing system with balking, reneging, and retention of reneged customers of single server M/M/1/N feedback (Bouchentouf et al., 2019; Bouchentouf & Guendouzi, 2020) is analyze and deals with variant multiple working vacations and impatience timers of Bernoulli feedback queueing system with that are depend on the server states. (Chang et al., 2018) give an analysis of an unreliable-server retrial queue is presented, along with customer feedback and impatience. The policies of truncated classical and constant retrial are considered. (Chowdhury & Rani, 2021) using the matrix-approach balking queue of catastrophic feedback studies and find steady-state solution. (Fackrell et al., 2021) In an M/M/1 feedback queue, where price and time-sensitive customers are homogeneous in terms of service valuation and cost per unit time of waiting, investigate the behaviour of equilibria. (Gupta, 2022) analyze Markovian queueing system with retention of reneged customers, setup time under feedback, Bernoulli schedule interruption, working vacation, and reneging of impatient customers. (Jain, 2016) presented with various schemes under a probability-based model. Analysis of M/G/1 feedback queue is also studied by (Saravanarajan & Chandrasekaran, 2014; Shanmuga Sundaram & Sivaram, 2021; Sundari & Srinivasan, 2012). Cost optimization analysis of a discrete-time finite-capacity multi-server queueing system with Bernoulli feedback, synchronous multiple and single working vacations, balking and reneging during both busy and working vacation periods studies by (Yahiaoui et al., 2019).

In this paper, a single server with a finite capacity queueing system. We check the performance analysis of the Markovian feedback queue with balking, reneging, and retention of the reneged customer. Customer dissatisfaction as a result of poor service quality is referred to as feedback in queueing literature. Customer retries service after receiving partial or incomplete service in the case of feedback. Model is described in section 2. The system's performance measures are discussed in section 3. Numerical and graphical system analysis is described in section 4 and 5. The paper concludes in section 6.

2. Model Description

Consider an M/M/1/N queue with instantaneous Bernoulli feedback with reneged customers and retention of reneged customers. The system's capacity is assumed to be finite. Customers arrive at the service station in a Poisson process with an arrival rate λ. There is a single server that serves all of the customers who arrive. Service times are exponential random variables with the parameter that are distributed independently and identically. After each service is completed, the customer can either join the queue at the end with a probability p_1 or leave with probability $q_1$where $p_1 + q_1 = 1$. Customers who have just arrived as well as those who have provided feedback are served in the order in which they join the tail end of the original queue. We don't differentiate between regular and feedback arrivals. Customers are served in a first-come, first-served fashion. When a service is unavailable for an extended period, the customer in line (regular arrival or feedback arrival) may become irritated. Each customer gets their timer when they arrive, which is based on an exponential distribution with parameter ξ. This is a customer's default time, after which he or she will either abandon the queue with probability $p_2(= 1 - q_2)$ or never return with complementary probability. With a high probability of balking, the arriving customer joins the system. An arriving customer is assumed to balk with probability n/N, where n is the number of customers in the system, and thus joins with probability $1 - \frac{n}{N}$, where N is the measure of the customer's willingness to join the queue. A continuous-time Markov chain is used to model a system of difference differential equations satisfied by the M/M/1/N feedback queue with reneged customers, balking, and retention of reneged customers.

Let $\{X(t): t \in \mathbb{R}^+\}$ be the number of customers in the system at time t. let $P_n(t) = P(X(t) = n), n = 0,1,2, \ldots, N$ be the state probability that there are n customers in the system at time t. Based on the above assumptions the steady-state equations of the model are

$$0 = -\lambda P_0 + \mu q_1 P_1; \qquad n = 0 \tag{1}$$

$$\begin{aligned} 0 = -\left\{\left(1 - \frac{n}{N}\right)\lambda + \mu q_1 + (n-1)\xi p_2\right\} P_n + (\mu q_1 + n\xi p_2) P_{n+1} \\ + \left(1 - \frac{N-1}{N}\right)\lambda P_{n-1}; \qquad 1 \le n \le N-1 \end{aligned} \tag{2}$$

$$0 = \left(1 - \frac{N-1}{N}\right)\lambda P_{N-1} - [\mu q_1 + (N-1)\xi p_2] P_N; \qquad n = N \tag{3}$$

After solving equation (1)-(3), we get

$$P_n = \prod_{r=1}^{n} \frac{N-(r-1)}{N} \frac{\lambda}{\mu q_1 + (r-1)\xi p_2} P_0 \tag{4}$$

Using the condition, $\sum_{n=0}^{N} P_n = 1$ we get

$$P_0 = \left[1 + \sum_{n=1}^{N} \prod_{r=1}^{n} \frac{N-(r-1)}{N} \frac{\lambda}{\mu q_1 + (r-1)\xi p_2}\right]^{-1} \tag{5}$$

3. Performance Measures

In this section, performance measures are derived.

$$L_s = \sum_{n=1}^{N} n \left[\prod_{r=1}^{n} \frac{N-(r-1)}{N} \frac{\lambda}{\mu q_1 + (r-1)\xi p_2}\right] P_0 \tag{6}$$

$$L_q = \sum_{n=1}^{N} n \left[\prod_{r=1}^{n} \frac{N-(r-1)}{N} \frac{\lambda}{\mu q_1 + (r-1)\xi p_2}\right] P_0 - \frac{\lambda}{\mu q_1} \tag{7}$$

$$W_s = \frac{1}{\lambda}\left\{\sum_{n=1}^{N} n \left[\prod_{r=1}^{n} \frac{N-(r-1)}{N} \frac{\lambda}{\mu q_1 + (r-1)\xi p_2}\right] P_0\right\} \tag{8}$$

$$W_q = \frac{1}{\lambda}\left\{\sum_{n=1}^{N} n \left[\prod_{r=1}^{n} \frac{N-(r-1)}{N} \frac{\lambda}{\mu q_1 + (r-1)\xi p_2}\right] P_0\right\} - \frac{1}{\mu q_1} \tag{9}$$

$$E(Customer\ Served) = \sum_{n=1}^{N} n\mu q_1 \left[\prod_{r=1}^{n} \frac{N-(r-1)}{N} \frac{\lambda}{\mu q_1 + (r-1)\xi p_2}\right] P_0 \tag{10}$$

Equation (6)-(10) denotes the expected number of customers in the system, in the queue, expected waiting time in the system, in the queue, and expected number of customers served respectively.

4. Sensitivity Analysis of Models

The numerical illustration is the subject of this section. The results of the sensitivity analysis are discussed. MATLAB and MS-Excel are used to calculate the results.

TABLE 1: WHERE $\mu = 2, \xi = 0.1, q_1 = 0.2, p_2 = 0.3$

λ	L_s	L_q	$E(s)$	W_s	W_q
1.0	2.8155	1.8629	1.1262	2.8155	1.8629
1.1	2.9733	2.0093	1.1893	2.7030	1.8266
1.2	3.1119	2.1396	1.2448	2.5933	1.7830
1.3	3.2342	2.2557	1.2937	2.4879	1.7351
1.4	3.3426	2.3594	1.3371	2.3876	1.6853
1.5	3.4391	2.4524	1.3757	2.2928	1.6350
1.6	3.5255	2.5361	1.4102	2.2034	1.5851
1.7	3.6031	2.6117	1.4413	2.1195	1.5363
1.8	3.6733	2.6802	1.4693	2.0407	1.4890
1.9	3.7368	2.7425	1.4947	1.9668	1.4434

Tables 1 is analysis of given model with variation in λ, μ, ξ, q_1, and p_2. The expected number of users in the system, the expected number of users in the queue, the expected number of users served $E(s)$, the expected waiting period in the system, and the expected waiting period in the queue. From table 1, we make the variation in arrival rate λ and take other parameters constant. This shows that the length of the queue and the system are increasing but waiting time in the queue and the system are decreasing. The expected number of customers is served increasingly.

TABLE 2: WHERE $\lambda = 2, \xi = 0.1, q_1 = 0.2, p_2 = 0.3$

μ	L_s	L_q	$E(s)$	W_s	W_q
1.0	3.3493	2.3672	2.0096	1.6746	1.1836
1.1	3.3062	2.3260	2.0499	1.6531	1.1630
1.2	3.2635	2.2854	2.0887	1.6318	1.1427
1.3	3.2212	2.2452	2.1260	1.6106	1.1226
1.4	3.1792	2.2055	2.1619	1.5896	1.1028
1.5	3.1377	2.1663	2.1964	1.5688	1.0832
1.6	3.0965	2.1276	2.2295	1.5482	1.0638
1.7	3.0557	2.0895	2.2612	1.5279	1.0447
1.8	3.0154	2.0518	2.2917	1.5077	1.0259
1.9	2.9754	2.0147	2.3208	1.4877	1.0073

From table 2, we increase the service rate μ, which results that the length of the queue and the system decreasing, the waiting time in the queue and the system is approximately reaching to same and customers are served increasingly.

TABLE 3: WHERE $\lambda = 2, \mu = 3, q_1 = 0.2, p_2 = 0.3$

ξ	L_s	L_q	$E(s)$	W_s	W_q
1.0	3.3493	2.3672	2.0096	1.6746	1.1836
1.1	3.1993	2.2209	1.9196	1.5996	1.1104

1.2	3.0672	2.0926	1.8403	1.5336	1.0463
1.3	2.9499	1.9790	1.7699	1.4750	0.9895
1.4	2.8450	1.8777	1.7070	1.4225	0.9388
1.5	2.7504	1.7868	1.6503	1.3752	0.8934
1.6	2.6648	1.7047	1.5989	1.3324	0.8523
1.7	2.5868	1.6301	1.5521	1.2934	0.8151
1.8	2.5155	1.5621	1.5093	1.2578	0.7811

From table 3, When we increase the reneging time all the performance 6 measures are decreasing.

TABLE 4: WHERE $\lambda = 2, \mu = 3, \xi = 0.1, p_2 = 0.3$

q_1	L_s	L_q	$E(s)$	W_s	W_q
1.0	4.0243	3.0261	1.2073	2.0122	1.5130
1.1	3.3493	2.3672	2.0096	1.6746	1.1836
1.2	2.7455	1.8037	2.4710	1.3728	0.9018
1.3	2.2478	1.3645	2.6974	1.1239	0.6823
1.4	1.8572	1.0399	2.7858	0.9286	0.5199
1.5	1.5563	0.8045	2.8014	0.7782	0.4022
1.6	1.3246	0.6335	2.7816	0.6623	0.3168
1.7	1.1443	0.5080	2.7462	0.5721	0.2540
1.8	1.0019	0.4142	2.7053	0.5010	0.2071

From table 4, When the parameter q_1 increases the length of the queue, the length of the system, and waiting time in the queue decreases. But customer served and the waiting time in the queue is increasing.

TABLE 5: WHERE $\lambda = 2, \mu = 3, \xi = 0.1, q_1 = 0.2$

p_2	L_s	L_q	$E(s)$	W_s	W_q
1.0	3.4612	2.4766	2.0767	1.7306	1.2383
1.1	3.4039	2.4206	2.0424	1.7020	1.2103
1.2	3.3493	2.3672	2.0096	1.6746	1.1836
1.3	3.2971	2.3162	1.9782	1.6485	1.1581
1.4	3.2471	2.2675	1.9483	1.6236	1.1337
1.5	3.1993	2.2209	1.9196	1.5996	1.1104
1.6	3.1534	2.1763	1.8921	1.5767	1.0881
1.7	3.1094	2.1336	1.8657	1.5547	1.0668
1.8	3.0672	2.0926	1.8403	1.5336	1.0463

From table 5, When the parameter p2 increases then all the performance measures are decreases. From the following figures, we can observe more clearly.

(i) *Effect on performance measures with respect to parameters:* Figure 1 shows that the performance measures derived from the above equations (6)-(10) with respect to given parameters λ, μ, ξ, q_1, and p_2. Figure 1(a) is performance measures Viruses arrival rate λ. In this figure, we make the variation in arrival rate λ and take other parameters constant. This shows that the length of the queue and the system are increasing but waiting time in the queue and the system are decreasing. The expected number of customers is served increasingly. Figure 1(b) is performance measures Viruses service rate μ. When we increase the service rate μ, it results that the length of the queue and the system decreasing, waiting time in the queue and the system approximately reaches to same and customers are served increasingly. Figure 1(c) is performance measures Viruses reneging time with parameter ξ. When we increase the reneging time all the performance measures are decreasing. Figure 1(d) is performance measures viruses not rejoin the queue with probability q_1. When the parameter q_1 increases the length of the queue, the length of the system, and waiting time in the queue decreases. But customer served and the waiting time in the queue is increasing. Figure 1(e) is performance measures viruses customers not retained in the queueing system with probability p_2. When this parameter increases then all the performance measures decrease.

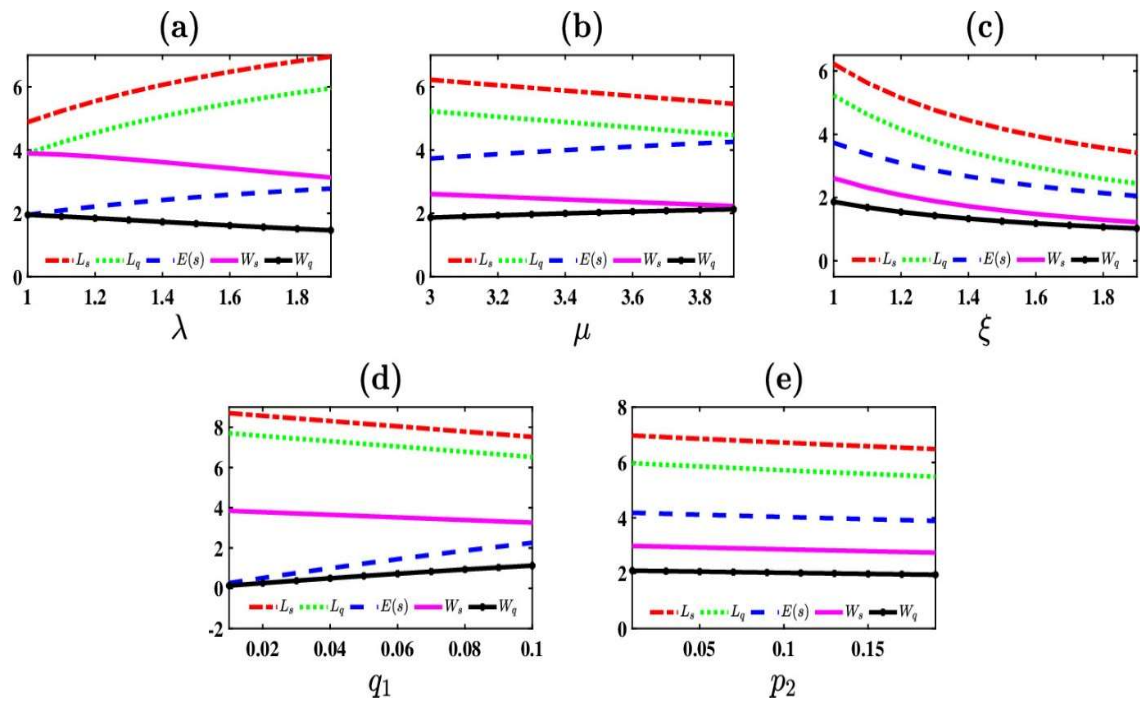

Figure 1: Performance Measures $L_s, L_q, E(s), W_s$ and W_q with respect to parameters (a) $\lambda = 1, 1.1, 1.2, \cdots, 1.9, \mu = 2, \xi = 0.1, q1 = 0.2, p2 = 0.3$ (b) $\mu = 3, 3.1, 3.2, \cdots, 3.9, \lambda = 2, \xi = 0.1, q1 = 0.2, p2 = 0.3$ (c) $\xi = 1, 1.1, 1.2, \cdots, 1.9, \lambda = 2, \mu = 3, q1 = 0.2, p2 = 0.3$ respectively.

(ii) *Effect on length of the system with respect to parameters*: Figure 2 represents the expected length of the system with respect to the parameters λ, μ, and ξ for different capacity of the system. Figure 2(a) shows that when the arrival rate λ increases length of the system is increasing. But we see that when the system capacity is N = 30 then the system's length is going high but slowly according to the system capacity N = 10 and N = 20. Figure 2(b) shows that when the service rate μ increasing length of the system is decreasing. But we see that when the system capacity is N = 30 then the system's length goes low but quickly according to the system capacity N = 10 and N = 20. Figure 2(c) shows that when the reneging time with parameter ξ increases length of the system is decreasing. But we see that when the system capacity is N = 30 then the system's length goes low but quickly according to the system capacity N = 10 and N = 20. Here, when system capacity is large then this model performs better.

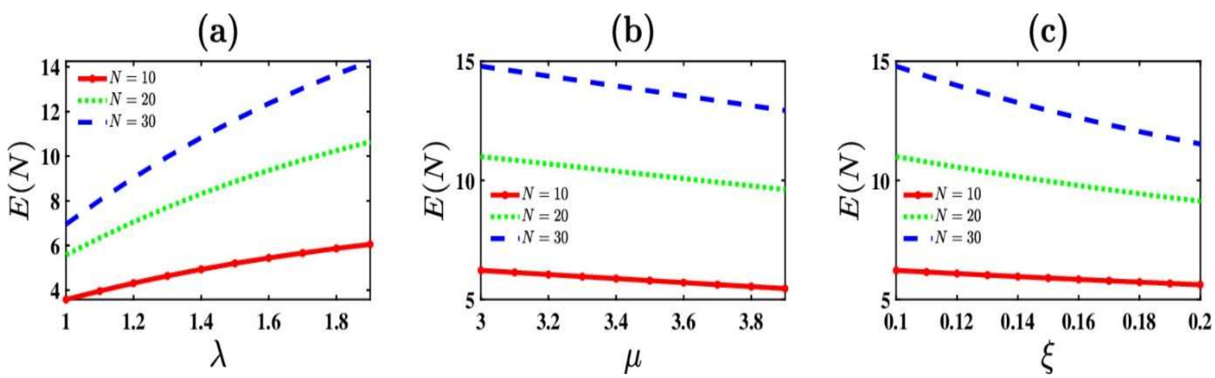

Figure 2: (a) Expected length of the system Vs arrival rate $\lambda, \mu = 3, \xi = 0.1, q_1 = 0.2, p2 = 0.3$, (b) Expected length of the system Vs service rate $\mu, \lambda = 2, \xi = 0.1, q_1 = 0.2, p_2 = 0.3$, and (c) Expected length of the system Vs rate of time $\gamma, \lambda = 2, \mu = 3, q_1 = 0.2, p_2 = 0.3$ for system capacity $N = 10, 20, 30$ respectively.

(iii) *Effect on queue length with respect to parameters:* Figure 3 represents the expected length of queue with respect to the parameters λ, μ, and ξ for different capacity of the system. Figure 3(a) shows that when the arrival rate λ increases length of the queue is increasing. But we see that when the system capacity is N = 30 then the queue length is going high but slowly according to the system capacity N = 10 and N = 20. Figure 3(b) shows that when the service rate μ increases length of the queue is decreasing. But we see that when the system capacity is N = 30 then the queue length is going low but quickly according to the system capacity N = 10 and N = 20. Figure 3(c) shows that when the reneging time with parameter ξ increases length of the queue is decreasing. But we see that when the system capacity is N = 30 then the queue length is going low but quickly according to the system capacity N = 10 and N = 20. Here, when system capacity is large then this model performs better.

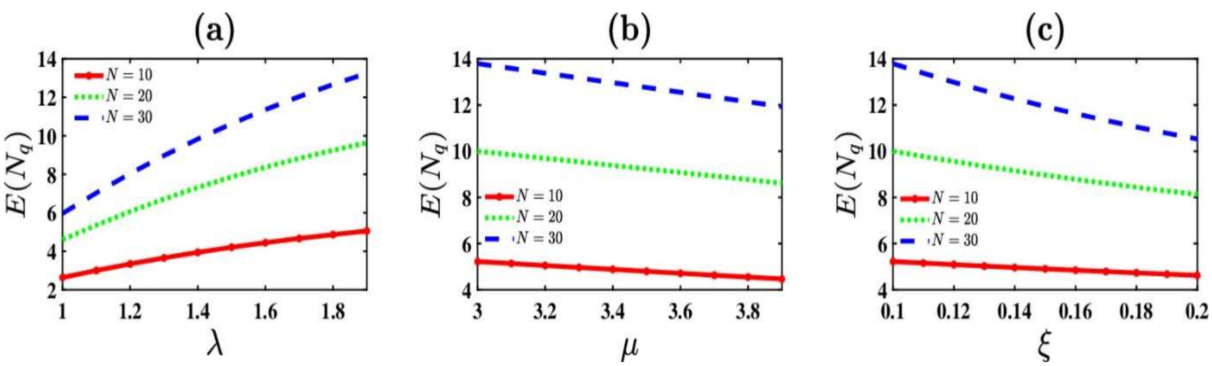

Figure 3: (a) Expected length of queue Vs arrival rate $\lambda, \mu = 3, \xi = 0.1, q_1 = 0.2, p_2 = 0.3$, (b) Expected length of queue Vs service rate $\mu, \lambda = 2, \xi = 0.1, q_1 = 0.2, p_2 = 0.3$, and (c) Expected length of queue Vs rate of time γ, $\lambda = 2, \mu = 3, q_1 = 0.2, p_2 = 0.3$ for system capacity $N = 10, 20, 30$ respectively.

(iv) *Effect on customer served with respect to parameters:* Figure 4 represents the expected Expected number of customers served with respect to the parameters λ, μ, and ξ for different capacity of the system. Figure 4(a) shows that when the arrival rate λ increases Expected number of customers served also increases. But we see that when the system capacity is N = 30 then the queue length is going high but slowly according to the system capacity N = 10 and N = 20. Figure 4(b) shows that when the service rate μ increasing Expected number of customers served is increasing. But we see that when the system capacity is N = 30 then the queue length is going high but quickly according to the system capacity N = 10 and N = 20. Figure 4(c) shows that when the reneging time with parameter ξ increases Expected the number of customers served is decreasing. But we see that when the system capacity is N = 30 then the Expected number of customers served is going low but quickly according to the system capacity N = 10 and N = 20. Here, when system capacity is large then this model performs better.

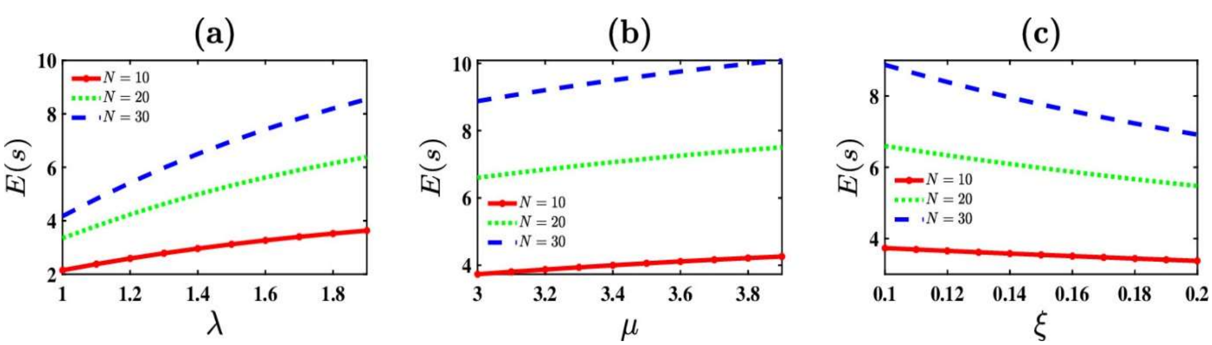

Figure 4: (a) Expected number of customers served Vs arrival rate $\lambda, \mu = 3, \xi = 0.1, q1 = 0.2, p2 = 0.3$, (b) Expected number of customer served Vs service rate $\mu, \lambda = 2, \xi = 0.1, q1 = 0.2, p2 = 0.3$, and (c) Expected number of customer served Vs rate of time $\gamma, \lambda = 2, \mu = 3, q1 = 0.2, p2 = 0.3$ for system capacity $N = 10, 20, 30$ respectively.

Conclusions

In this paper, we investigate an $M/M/1/N$ feedback queuing model with reneging, balking, and reneged customer retention. The model's steady-state solution is discovered, as well as some performance measures. The model's findings could be useful in simulating a variety of production and service processes that involve feedback and impatient customers. The analysis of the model is limited to finite capacity. The model's infinite capacity case can also be investigated. Furthermore, the model can be solved in a transient state to obtain time-dependent results.

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