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IMPACT OF BI-LINEAR TEMPERATURE VARIATION ON THE VIBRATION OF ISOTROPIC VISCO-ELASTIC SQUARE PLATES C-C-C-C WITH CIRCULAR THICKNESS VARIATION

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ABSTRACT

The dynamic response of isotropic viscoelastic square plates to temperature variations is a critical area of study in structural mechanics and materials science. This research paper investigates the influence of bi-linear temperature distributions on the vibration characteristics of such plates. Through analytical formulations and numerical simulations, the paper explores how abrupt changes in temperature across the plate's thickness affect its natural frequencies, mode shapes, and damping properties. The findings provide insights into optimizing the design and performance of viscoelastic structures under varying thermal conditions.

KEYWORDS: temperature, plate, vibration, frequency, thickness.

INTRODUCTION

In the realm of structural mechanics and materials science, understanding the dynamic response of isotropic viscoelastic square plates under varying thermal conditions is crucial for optimizing the performance and durability of engineering structures. Isotropic viscoelastic materials exhibit both elastic and viscous behaviors under mechanical stress, making them suitable for applications where damping and deformation over time are critical factors. Square plates, as fundamental components in aerospace, automotive, and civil engineering, are particularly sensitive to environmental factors such as temperature fluctuations, which can significantly influence their vibrational characteristics.

The focus of this research paper is to investigate the effect of bi-linear temperature variations on the vibration behavior of isotropic viscoelastic square plates with circular thickness profiles. Bi-linear temperature distributions involve abrupt changes in temperature gradients across the thickness of the plate, which introduce non-linearities in material properties such as stiffness and damping coefficients. Understanding these effects is essential for predicting how structural dynamics are affected under realistic operational conditions.

The study employs a multidisciplinary approach, integrating principles from structural dynamics, viscoelasticity, and thermal physics. Analytical formulations and numerical simulations are utilized to model the coupled behavior of temperature gradients and mechanical responses in isotropic viscoelastic materials. By developing a comprehensive understanding of these interactions, the

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research aims to elucidate how bi-linear temperature variations influence natural frequencies, mode shapes, and damping ratios of square plates.
Free vibration of visco-elastic orthotropic rectangular plates was discussed by Sobotka [1]. Gupta and Khanna [2] discussed vibration of viscoelastic rectangular plate with linearly thickness variations in both directions. Leissa's monograph [3] contains an excellent discussion of the subject of vibrating plates with elastic edge support. Several authors [4,5] have studied the thermal effect on vibration of homogeneous plates of variable tion on non-homogeneous rectangular plates of varying thickness. Tomar and Gupta [6-8] solved the vibration problem of orthotropic rectangular plate of varying thickness subjected to a thermal gradient. Gupta, Lal and Sharma [9] discussed the vibration of non-homogeneous circular plate of nonlinear thickness variation by a quadrature method. Gupta, Johri and Vats [10] solved the problem of thermal effect on vibration of non-homogeneous orthotropic rectangular plate having bi-directional parabolically varying thickness. Gupta, Kumar and Gupta [11] studied the vibration of visco-elastic orthotropic parallelogram plate with a linear variation of thickness. Recently, Gupta and Kumar [12] solved the vibration problem of non-homogeneous visco-elastic rectangular plate of linearly varying thickness subjected to linearly thermal effect. The research methodology involves theoretical derivations based on fundamental equations governing the motion of viscoelastic structures under thermal gradients. These derivations are complemented by finite element simulations, which provide detailed insights into the complex interactions between temperature distributions and mechanical responses. Experimental validation through modal analysis techniques further corroborates the theoretical predictions, ensuring the reliability and accuracy of the developed models.

By advancing our understanding of how bi-linear temperature variations influence the vibration characteristics of isotropic viscoelastic square plates, this research contributes to the broader field of materials science and engineering. It provides a foundation for developing improved design guidelines and methodologies aimed at enhancing the performance and resilience of structural components under diverse environmental conditions. Ultimately, the insights gained from this study pave the way for more efficient and sustainable engineering practices across various industries.

EQUATION OF MOTION

Differential equation of transverse motion of a visco-elastic plate of variable thickness in Cartesian co-ordinates [1]:

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{yx}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \rho h \frac{\partial^2 w}{\partial t^2} \qquad (1)$$

The expression for M_x, M_y, M_{yx} are given by

$$\left.\begin{aligned} M_x &= -\widetilde{D} D_1 \left(\frac{\partial^2 w}{\partial x^2} + \vartheta \frac{\partial^2 w}{\partial y^2}\right) \\ M_y &= -\widetilde{D} D_1 \left(\frac{\partial^2 w}{\partial y^2} + \vartheta \frac{\partial^2 w}{\partial x^2}\right) \\ M_{yx} &= -\widetilde{D} D_1 (1-\vartheta) \frac{\partial^2 w}{\partial y \partial x} \end{aligned}\right\} \qquad (2)$$

where $\widetilde{D}$ is visco-elastic operator.

On substitution the values M_x, M_y and M_{yx} from equation (2) in (1) and taking w, as a product of two function, equal to w(x,y,t)=W(x,y)T(t), equation (1) become:

$$\left[\begin{array}{l} D_1\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}+\frac{\partial^3 W}{\partial x\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3}+\frac{\partial^3 W}{\partial x^2\partial y}\right) \\ +\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2}+\vartheta\frac{\partial^2 W}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\vartheta\frac{\partial^2 W}{\partial x^2}\right)+2(1-\vartheta)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y} \end{array}\right] \Big/ \rho h W = -\frac{\ddot{T}}{\tilde{D}T} \qquad (3)$$

Here dot denote differentiation with respect to t, taking both sides of equation (3) are equal to a constant p^2 (square of frequency), we have

$$\Big[\quad D_1\left(W_{,xxxx}+2W_{,xxyy}\right)-2D_{1,x}\left(W_{,xxx}+W_{,xyy}\right)+2D_{1,y}\left(W_{,yyy}+W_{,yxx}\right)+D_{1,xx}\left(W_{,xx}+\vartheta W_{,yy}\right)+D_{1,yy}\left(W_{,yy}+\vartheta W_{,xx}\right)$$
$$+2(1-\vartheta)D_{1,xy}W_{,x} \qquad (4)$$

is a differential equation of transverse motion for non-homogeneous plate of variable thickness. Here, D_1 is the flexural rigidity of plate i.e.

$$D_1 = \frac{Eh^3}{12(1-\vartheta^2)} \qquad (5)$$

and corresponding two-term deflection function is taken as [5]

$$W = \left[\left(\frac{x}{a}\right)\left(\frac{y}{a}\right)\left(1-\frac{x}{a}\right)(1-y/a)\right]^2\left[A_1 + A_2\left(\frac{x}{a}\right)\left(\frac{y}{a}\right)\left(1-\frac{x}{a}\right)(1-y/a)\right] \qquad (6)$$

In the above equation A_1 and A_2 are constants satisfy boundary conditions.

MATHEMATICAL ASSUMPTIONS

It is assumed that temperature varies parabolically in two directions i.e.

$$\tau = \tau_0(1-x/a)(1-y/a) \quad (7)$$

where τ denotes the temperature excess above the reference temperature at any point on the plate and τ_0 denotes the temperature at any point on the boundary of plate and "a" is the length of a side of square plate. The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed in this

$$E = E_0(1-\gamma\tau) \qquad (8)$$

where, E_0 is the value of the Young's modulus at reference temperature i.e. $\tau = 0$ and γ is the slope of the variation of E with τ. The modulus variation (5) become

$$E = E_0[1-\alpha(1-x/a)(1-y/a)] \qquad (9)$$

where $\alpha = \gamma\tau_0 (0 \le \alpha < 1)$, thermal gradient.

It is assumed that thickness also varies parabolic in x and y directions as shown below:

$$h = h_0\left(1+\beta_1(1-\sqrt{1-\frac{x^2}{a^2}})\right)\left(1+\beta_2(1-\sqrt{1-\frac{y^2}{a^2}})\right) \qquad (10)$$

where β_1 is taper parameters in x- directions respectively and $h=h_0$ at x=y=0. Put the value of E & h from equation (9) & (10) in the equation (5), one obtain

$$D_1 = \frac{\left[E_0[1-\alpha(1-x/y)(1-y/x)]h_0\left(1+\beta_1(1-\sqrt{1-\frac{x^2}{a^2}})\right)\left(1+\beta_2(1-\sqrt{1-\frac{y^2}{a^2}})\right)\right]}{12(1-\vartheta^2)} \qquad (11)$$

Rayleigh-Ritz technique is applied to solve the frequency equation. In this method, one requires maximum strain energy must be equal to the maximum kinetic energy. So it is necessary for the problem under consideration that

$$\delta(K^* - S^*) = 0 \qquad (12)$$

for arbitrary variations of W satisfying relevant geometrical boundary conditions. Since the plate is assumed as clamped at all the four edges, so the boundary conditions are:

$$\left.\begin{aligned} W = W_{,x} = 0, \qquad x = 0, a \\ W = W_{,y} = 0, \qquad y = 0, a \end{aligned}\right\} \qquad (13)$$

Now assuming the non-dimensional variables as

$$X = \frac{x}{a},\ Y = \frac{y}{a},\ \ \overline{W} = \frac{W}{a},\ \ \overline{h} = \frac{h}{a} \qquad (14)$$

The kinetic energy K* and strain energy S* are [2]

$$K^* = \left(\frac{1}{2}\right)\rho p^2 \overline{h_0} a^5 \int_0^1 \int_0^1 \left[\left(1 + \beta_1(1 - \sqrt{1 - X^2})\right)\left(1 + \beta_2(1 - \sqrt{1 - Y^2})\right) \ \ \overline{W}^2\ \right] dYdX \qquad (15)$$

and

$$S^* = Q \int_0^1 \int_0^1 \Bigg[[1 - \alpha(1 - X)(1 - Y)]\left[\left(1 + \beta_1(1 - \sqrt{1 - X^2})\right)\left(1 + \beta_2(1 - \sqrt{1 - Y^2})\right)\right]^3 \left\{\left(\overline{W}_{,XX}\right)^2 + \left(\overline{W}_{,YY}\right)^2 + 2\vartheta \overline{W}_{,XX}\overline{W}_{,YY} + 2(1 - \vartheta)\left(\overline{W}_{,XY}\right)^2\right\}\Bigg] dYdX \qquad (16)$$

where, $Q = \frac{E_0 {h_0}^3 a^3}{24(1-\vartheta^2)}$

Using equations (15) & (16) in equation (12), one get

$$(S^{**} - \lambda^2 K^{**}) = 0 \qquad (17)$$

where,

$$S^{**} = \int_0^1 \int_0^1 \Bigg[[1 - \alpha(1 - X)(1 - Y)]\left[\left(1 + \beta_1(1 - \sqrt{1 - X^2})\right)\left(1 + \beta_2(1 - \sqrt{1 - Y^2})\right)\right]^3 \left\{\left(\overline{W}_{,XX}\right)^2 + \left(\overline{W}_{,YY}\right)^2 + 2\vartheta \overline{W}_{,XX}\overline{W}_{,YY} + 2(1 - \vartheta)\left(\overline{W}_{,XY}\right)^2\right\}\Bigg] dYdX \qquad (18)$$

and

$$K^{**} = \int_0^1 \int_0^1 \left[\left(1 + \beta_1(1 - \sqrt{1 - X^2})\right)\left(1 + \beta_2(1 - \sqrt{1 - Y^2})\right) \quad \overline{W^2}\right] dYdX \qquad (19)$$

Here, $\lambda^2 = 12\frac{\rho p^2(1-\vartheta^2)a^2}{E_0 {h_0}^2}$ is a frequency parameter.

Equation (19) consists two unknown constants i.e. A_1 & A_2 arising due to the substitution of W. These two constants are to be determined as follows:

$$\frac{\partial(S^{**} - \lambda^2 K^{**})}{\partial A_n}, = 0 \quad , n=1, 2 \qquad (20)$$

On simplifying (20), we gets

$$b_{n1}A_1 + b_{n2}A_2 = 0 \quad , n=1, 2 \qquad (21)$$

where bn_1, bn_2 (n=1,2) involve parametric constant and the frequency parameter.
For a non-trivial solution, the determinant of the coefficient of equation (21) must be zero. So one gets, the frequency equation as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} \qquad (22)$$

With the help of equation (22), one can obtain a quadratic equation in λ^2 from which the two values of λ^2 can found. These two values represent the two modes of vibration of frequency i.e. λ_1 (Mode1) & λ_2 (Mode2) for different values of taper constant and thermal gradient for a clamped plate.

RESULTS AND DISCUSSIONS

Here all the estimations have been accomplished for frequency of visco- elastic square plate for uncommon estimations of decrease constants of taper constants and thermal gradient for various focuses for first two modes of vibrations had been ascertained numerically.

Figure I:- In this fig. I we can see that the value of frequency increasing in both the modes of vibration when we increasing value of thermal effect α from 0.0 to 1.0.

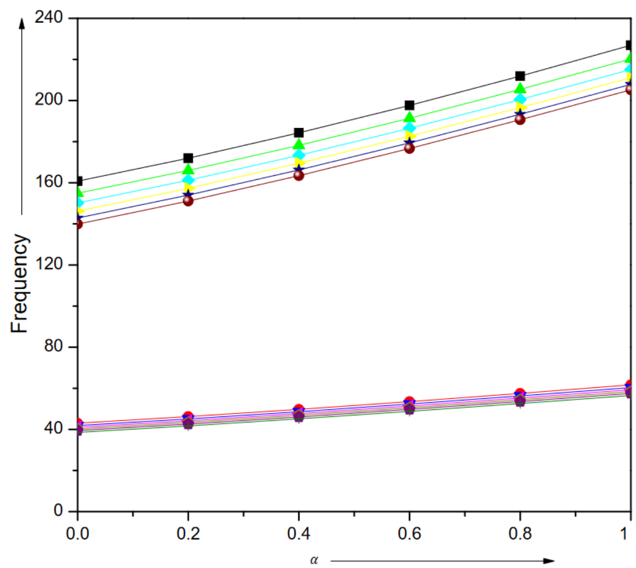

Fig. I. Frequency Vs Thermal Gradient α

Figure II:- In this fig. II we can see that the value of frequency decreasing in both the modes of vibration when we increasing value of taper constant β_1 from 0.0 to 1.0.

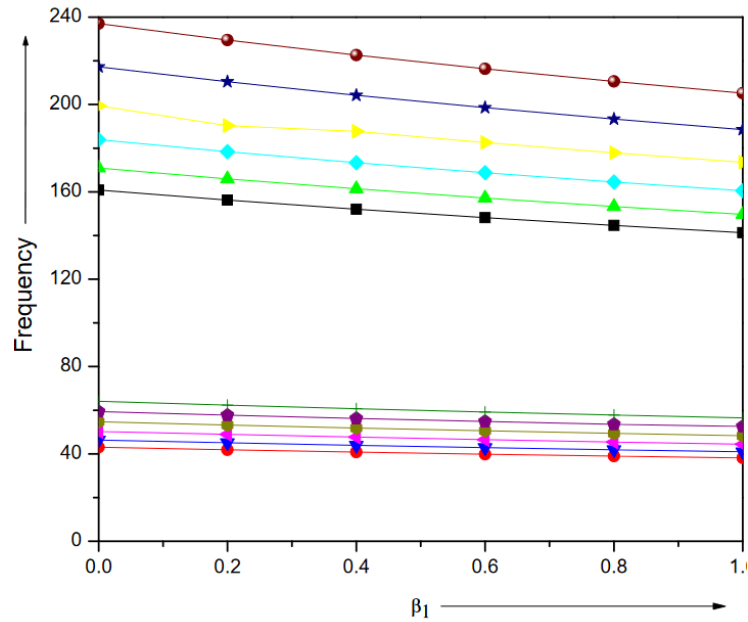

Fig. II. Frequency Vs Taper Constant β_1

Figure III:- In this fig. III we can see that the value of frequency decreasing in both the modes of vibration when we increasing value of taper constant β_2 from 0.0 to 1.0.

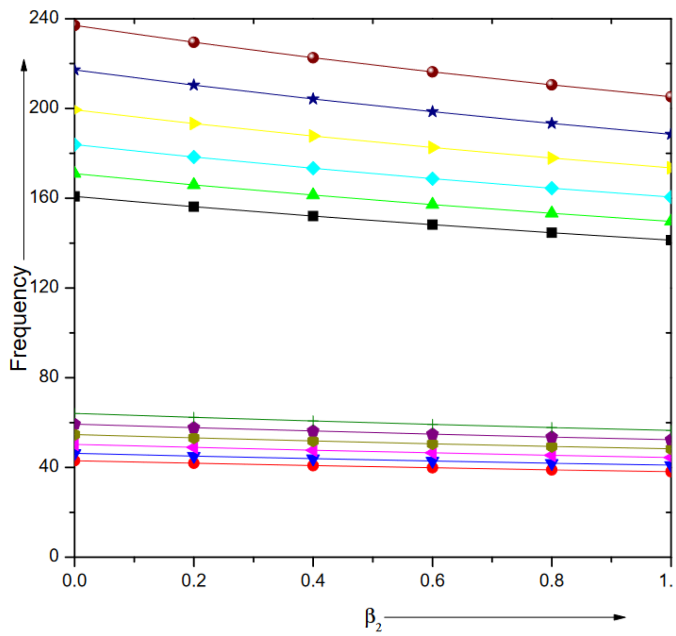

Fig. III. Frequency Vs Taper Constant β_2

CONCLUSION

In conclusion, the study highlights the critical importance of accounting for bi-linear temperature variation in the analysis of isotropic visco-elastic plates with circular thickness variation. The dynamic behavior of these plates is significantly influenced by temperature changes, which affect both the natural frequencies and damping properties. This comprehensive understanding aids in the design of robust structures capable of withstanding varied thermal conditions while maintaining desired vibrational performance.

REFERENCES

[1]. Z. Sobotka, "Free Vibration of Visco-Elastic Orthotropic Rectangular Plates," Acta Technica, Vol. 23, No. 6, 1978, pp. 678-705.

[2]. A. K. Gupta and A. Khanna, "Vibration of Viscoelastic Rectangular Plate with Linearly Thickness Variations in Both Directions," Journal of Sound and Vibration, Vol. 301, No. 3-5, 2007, pp. 450-457.

[3]. A. W. Leissa, "Vibration of Plate," NASA SP-60, 1969.

[4]. S. R. Li and Y. H. Zhou, "Shooting Method for Non Lin ear Vibration and Thermal Buckling of Heated Orthotropic Circular Plates," Journal of Sound and Vibration, Vol. 248, No. 2, 2001, pp. 379-386.

[5]. D. V. Bambill, C. A. Rossil, P. A. A. Laura and R. E. Rossi, "Transverse Vibrations of an Orthotropic Rectangular Plate of Linearly Varying Thickness and with a Free Edge," Journal of Sound and Vibration, Vol. 235, No. 3, 2000, pp. 530-538.

[6]. J. S. Tomar and A. K. Gupta, "Effect of Thermal Gradient on Frequencies of Orthotropic Rectangular Plate Whose Thickness Varies in Two Directions," Journal of Sound and Vibration, Vol. 98, No. 2, 1985, pp. 257-262.

[7]. J. S. Tomar and A. K. Gupta, "Thermal Effect on Fre quencies of an Orthotropic Rectangular Plate of Linearly Varying Thickness," Journal of Sound and Vibration, Vol. 90, No. 3, 1983, pp. 325-331.

[8]. J. S. Tomar and A. K. Gupta, "Effect of Exponential Temperature Variation on Frequencies of an Orthotropic Rectangular Plate of Exponentially Varying Thickness," Proceeding of the Workshop on Computer Application in Continum Mechanics, Roorkee, 11-13 March 1986, pp.183-188.

[9]. U. S. Gupta, R. Lal and S. Sharma, "Vibration Analysis of Non-Homogenous Circular Plate of Nonlinear Thickness Variation by Differential Quadrature Method," Journal of Sound and Vibration, Vol. 298, No. 4-5, 2006, pp. 892-906.

[10]. A. K. Gupta, T. Johri and R. P. Vats, "Thermal Effect on Vibration of Non-Homogeneous Orthotropic RectangularnPlate Having Bi-Directional Parabolically Varying Thick ness," Proceeding of International Conference in World Congress on Engineering and Computer Science 2007 (WCECS 2007), San Francisco, 24-26 October 2007, pp.784-787.

[11]. A. K. Gupta, A. Kumar and D. V. Gupta, "Vibration of Visco-Elastic Orthotropic Parallelogram Plate with Linearly Thickness Variation," Proceeding of International Conference in World Congress on Engineering and Computer Science 2007 (WCECS 2007), San Francisco,24-26 October 2007, pp. 800-803.

[12]. A. K. Gupta and L. Kumar, "Thermal Effect on Vibration of Non-Homogenous Visco-Elastic Rectangular Plate of Linear Varying Thickness," Meccanica, Vol. 43, No. 1, 2008, pp. 47-54.