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LYAPUNOV EXPONENT AND CORRELATION DIMENSION ANALYSIS OF TIME SERIES DATA OF ATMOSPHERIC TEMPERATURE

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Abstract

Calculations of Lyapunov exponent and correlation dimension are popular methods to identify chaotic behavior of a nonlinear dynamical system. In these methods time series data of a single dynamical variable of the system is used. Here we have taken atmosphere as a nonlinear dynamical system and atmospheric temperature as a single dynamical variable. The objective of this study is to analyze chaotic behavior of time series data of daily atmospheric temperature of Delhi during dry season from 1995 to 2019. We have calculated the positive value of Lyapunov exponent and non-integral value of correlation dimensions 0.000397& 0.90860 respectively, which indicates the presence of chaotic behavior in the atmosphere. This study would help us in analyzing the chaotic nature of atmosphere based on temperatures well as other nonlinear dynamical variables existing in atmosphere.

Keywords: Nonlinear Dynamics, Chaos, Attractor, Lyapunov Exponent, Correlation Dimension, Time series data

1. Introduction

Climate models are nonlinear dynamical systems. A model is a prototype of a larger system may numerical or mathematical model. It may be used for further studies of its characteristics. The climate models are Environmental Interfaces Water-Air Interface. A Nonlinear dynamical system is one whose time evolution equations are nonlinear in their dynamical variables. Chaos is exhibited by nonlinear dynamical systems. Nonlinear dynamical systems are those whose governing differential equations are nonlinear. When trajectories of nonlinear dynamical system in state space unpredictably change even for a very small change in initial condition, the system exhibits chaos. When trajectories evolve for a long time in state space, they form a geometrical structure and it appears that the trajectories attract towards the structure. This structure is called attractor. Chaotic dynamical systems are ubiquitous in nature. Weather and climate usually show chaotic behavior. [1][2][3][13][15]

Poincare is believed as the first person who studied chaos in latter 19th century. In 1963, Lorenz revealed the chaotic nature of weather, which is known as butterfly effect. The formal use of chaos has been started since 1975 by Li and Yorke. [3]Today, chaos is widely studied and the concept of quantifiers of chaos have been introduced to observe chaotic behavior in time series data.[2][3] Temperature time series data are usually nonlinear and complex.[3]

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Using Lyapunov exponent, Poincare map and Hurst exponent as quantifiers of chaos, A. Antony Suresh and R. Samuel Selvaraj analyzed chaotic behavior in daily mean air temperature and relative humidity data for Chennai, India.[4]In Nigeria, A. T. Adewole, E. O. Falayi, T. O. Roy-Layinde, A. D. Adelaja have applied Lyapunov exponent, phase space reconstruction, actual mutual information (AMI) and false nearest neighbors (FNNs) as the tools to analyze chaotic behavior in time series data of air temperature, relative humidity and wind speed.[5]Chaotic behavior of a system can be observed from the time series data of a single variable of the system by calculating Lyapunov exponent[6][7][12] and correlation dimension.[2][3]Minimum embedding space dimension and Reconstruction of phase has also been used as an analytical tool to observe presence of chaos in time series data of temperature.[8][9]

The aim of our study is observation of chaotic behavior in temperature time series data of Delhi, India by calculating **Lyapunov exponent** and **correlation dimension.**

2. Theoretical Analysis

2.1 Lyapunov exponent

Lyapunov exponent is the measure of divergence of nearby trajectories in state space. A positive value of Lyapunov exponent indicates the presence of chaos. Lyapunov exponent is calculated from time series data of a single dynamical variable of a non-linear dynamical system.Lyapunov Exponent will be calculated by drawing plots between $\mathbf{log}\frac{\boldsymbol{d_n}}{\boldsymbol{d_0}}$ vs $\boldsymbol{n}$.Slope will give Lyapunov exponent.

Let x_0, x_1, x_2, x_3 ……………….. denote time series data and if we choose x_i and x_j from the sequence, which are close to each other, then

$$d_0 = |x_j - x_i|$$
$$d_1 = |x_{j+1} - x_{i+1}|$$
$$d_2 = |x_{j+2} - x_{i+2}|$$

$$d_n = |x_{j+n} - x_{i+n}|$$

The Lyapunov exponent λ is calculated from the following formula,

$$\lambda = \frac{1}{n}\, ln\frac{d_n}{d_0}$$

As, we can see the value of λ depends upon the initial value x_i, so, for a large number of values of $x_i's$ (practically 30 to 40), average value of λ is calculated. If time series data or system is chaotic, λ is expected to come positive.[2][3][6][7][12][14]

2.2 Correlation Dimension

Grassberger and Proccacia introduced the concept of correlation dimension, which is deduced from the correlation sum. To calculate correlation sum, we let trajectory evolve for a long time on an attractor. We collect N trajectory points as data. Then for each point i on the trajectory, we look for the number of trajectory points lying within the distance R of the point i, excluding the point i itself. We are denoting this number $N_i(R)$. Now, we define $p_i(R)$ as the relative number of points within the distance R of the i^{th} point;

$p_i(R) = \frac{N_i}{N-1}$

Then, correlation sum, $C(R)$ is calculated as,

$$C(R) = \frac{1}{N}\sum_{I=1}^{N} p_i(R)$$

With the help of **Heaviside step function, $\boldsymbol{\Theta(x)}$,** we modified the formula as,

$$p_i(R) = \frac{1}{N-1}\sum_{i=1}^{N}\sum_{i\neq j,j=1}^{N} \boldsymbol{\Theta}(R - |x_i - x_j|)$$

Then,

$$\boldsymbol{C(R) = \frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{i\neq j,j=1}^{N} \Theta(R - |x_i - x_j|)}$$

To characterize the entire attractor, we take limit $N \to \infty$, the correlation dimension D_C is defined as

$$C(R) = \lim_{R\to 0} kR^{D_C}$$

Taking logarithm both sides,

$$D_C = \lim_{R\to 0}\frac{\log C(R)}{\log R}$$

So, the slope of $\log C(R) \sim \log R$ gives the correlation dimension D_C.[2][3]

3. Graphical Analysis

We have chosen daily mean temperature data of Delhi during dry season from 10 April to 10 June every year from 1995 to 2019. The source of data is secondary, which has been retrieved from the link**https://academic.udayton.edu/kissock/http/Weather/gsod95-current/INDELHI.txt** . Some data is missing. During dry season heat wave is one of the main causes of deaths, soforecasting of temperature would help in planning the strategy to encounter such a situation. Here we have applied the theory of chaotic time series analysis to detect only the chaotic behavior in time series data of temperature.

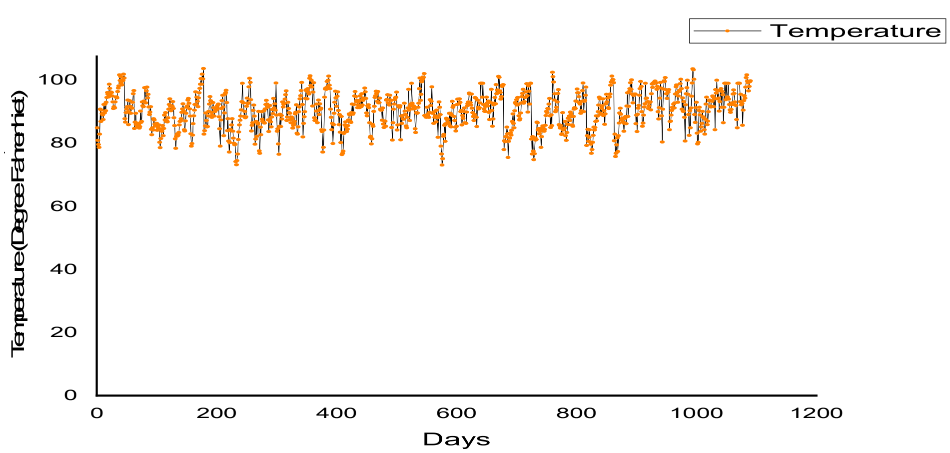
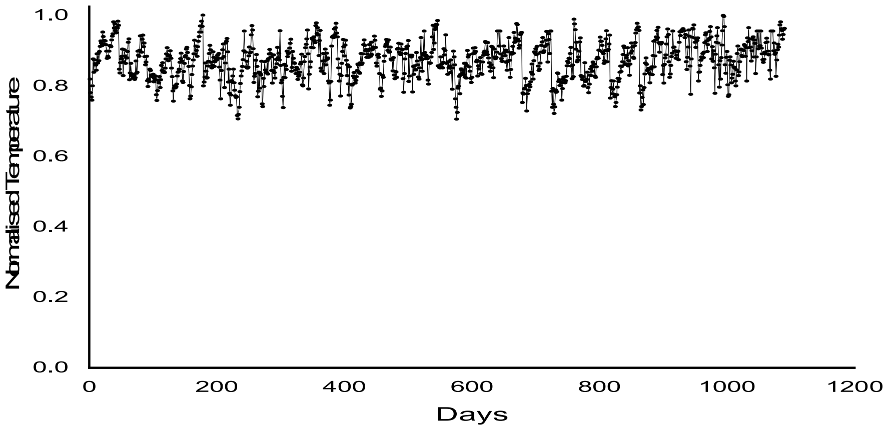


Figure-1 isthe plot for daily temperature data of Delhi from the year 1995 to 2019during the month of April & June ~ Days.



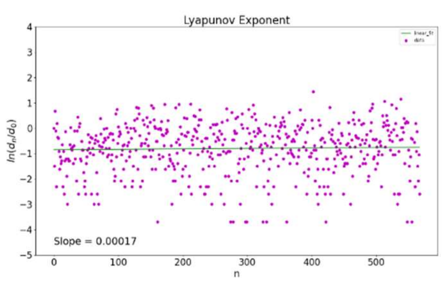
(Figure-2)

Figure-2 is the plot is for Normalized value of temperature ~ Days.

We have drawn plots choosing 30 values of $x_i's$and $x_j's$. Slopes of the plots are Lyapunov exponents(λ's). We have also drawn plots between 30 values of Lyapunov exponents (λ's) and their corresponding lags to show some of Lyapunov exponents are also negative, but their average over all thirty values is positive. Again, we have drawn a plot ofln $C(R)$~ ln $(R)$to know its slope, which is correlation dimension.

3.1Graphical presentation of Lyapunov exponents (λ's)

From the normalized data of temperature two sets $x_i's\ and\ x_j's$have been prepared with different lags and different number of data points n in each set (n isfor the number of $x_i's\ or\ x_j's$),d_0 is for the difference between initial value of $x_i\ and\ x_j$ and d_n is for the difference between n[th] values of x_i and x_j.



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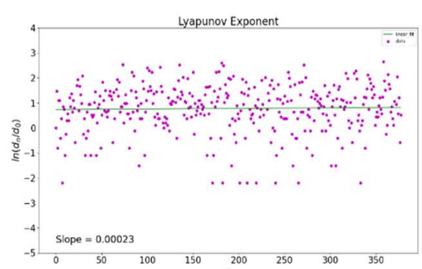

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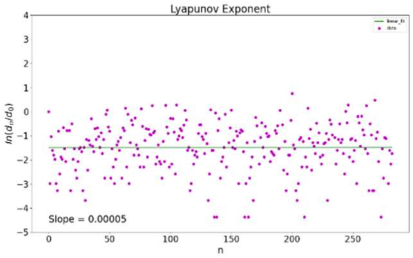

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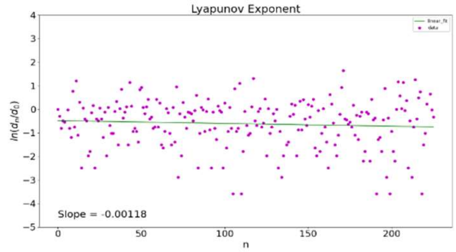

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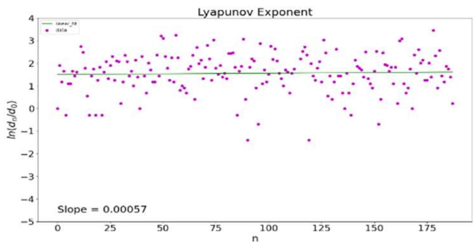

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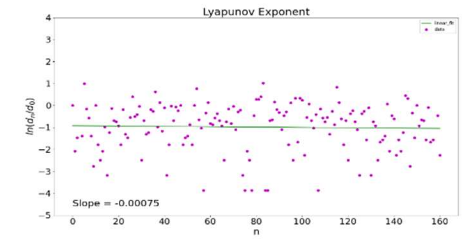

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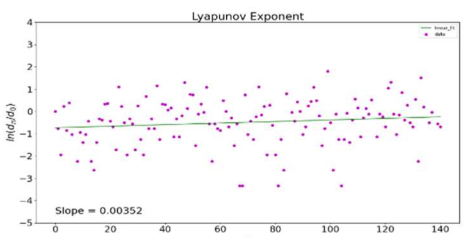

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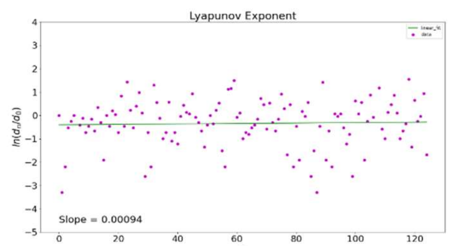

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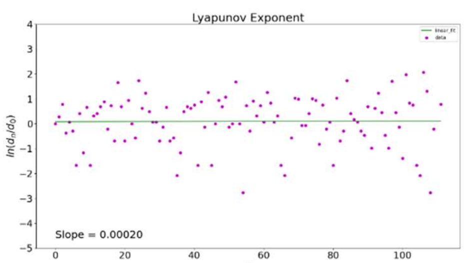

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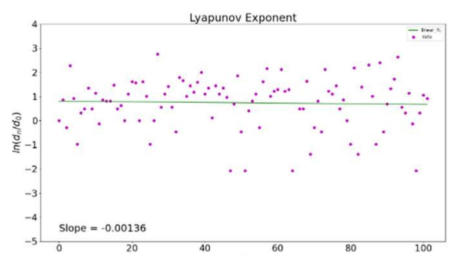

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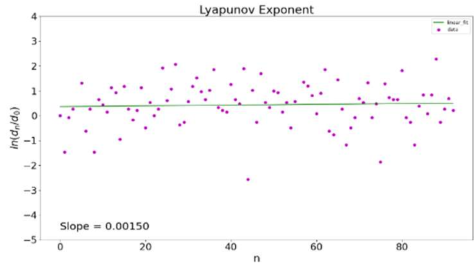

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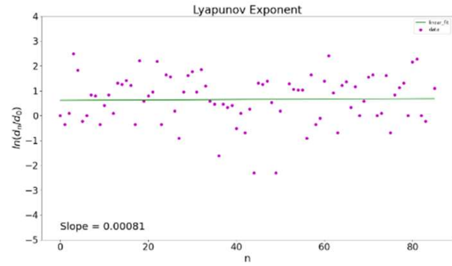

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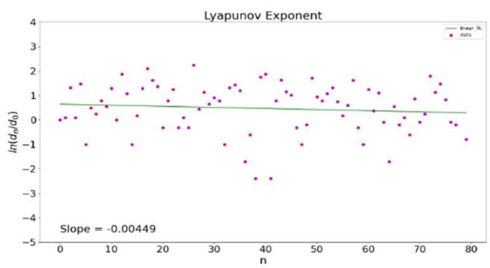

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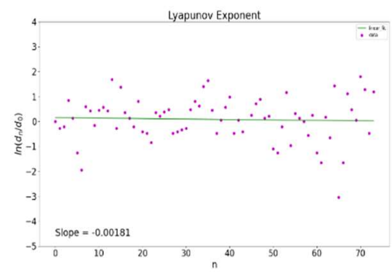

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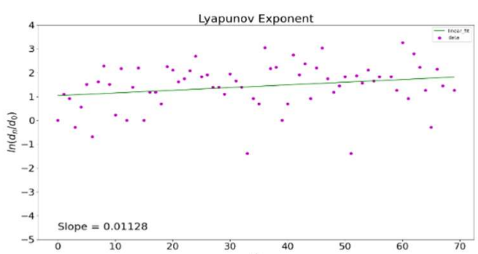

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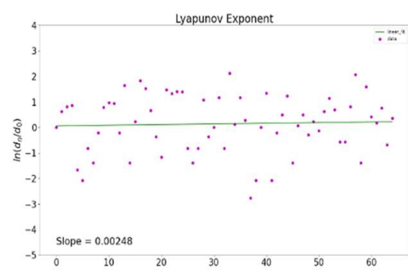

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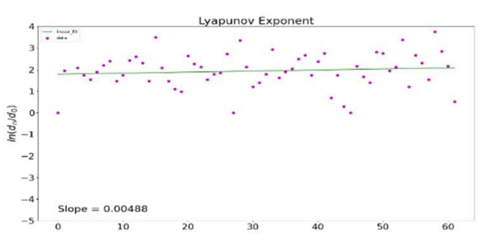

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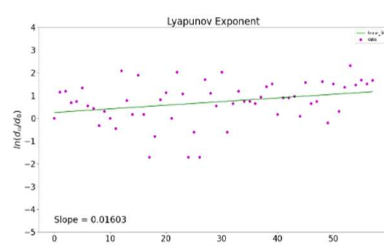

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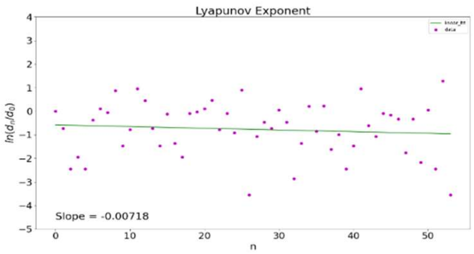

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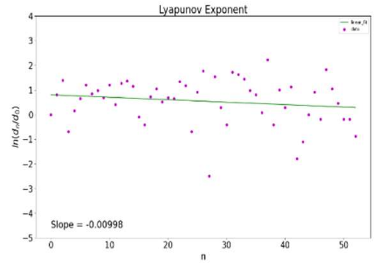

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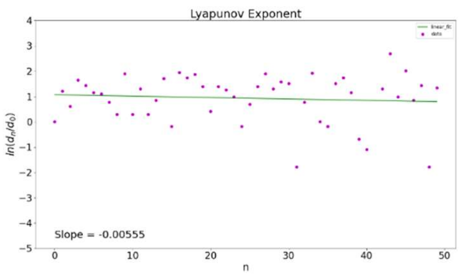

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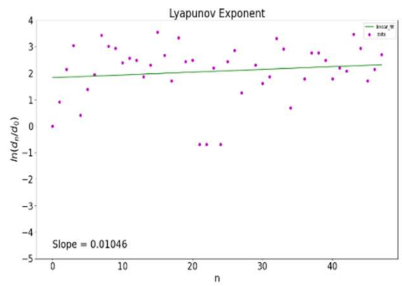

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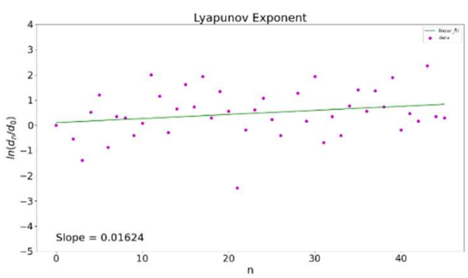

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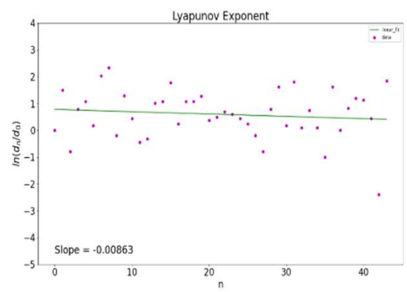

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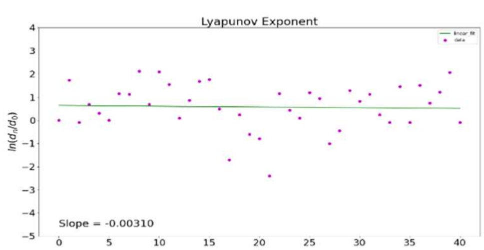

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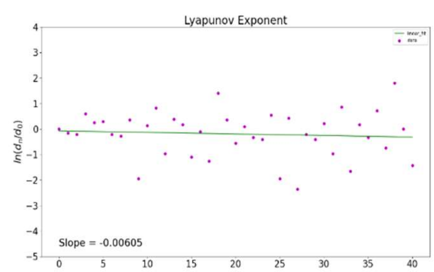

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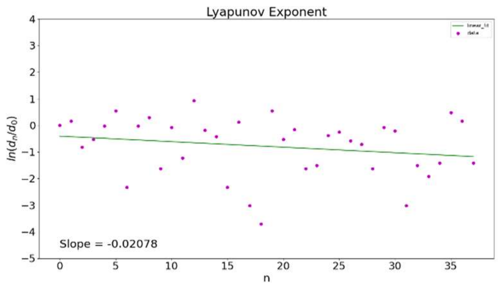

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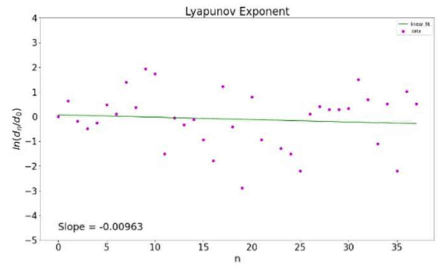

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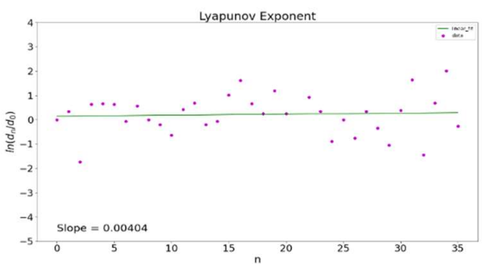

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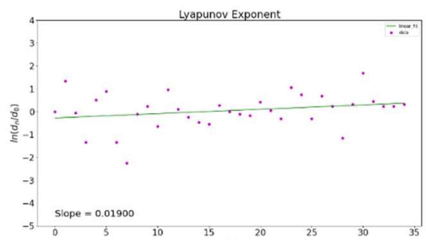

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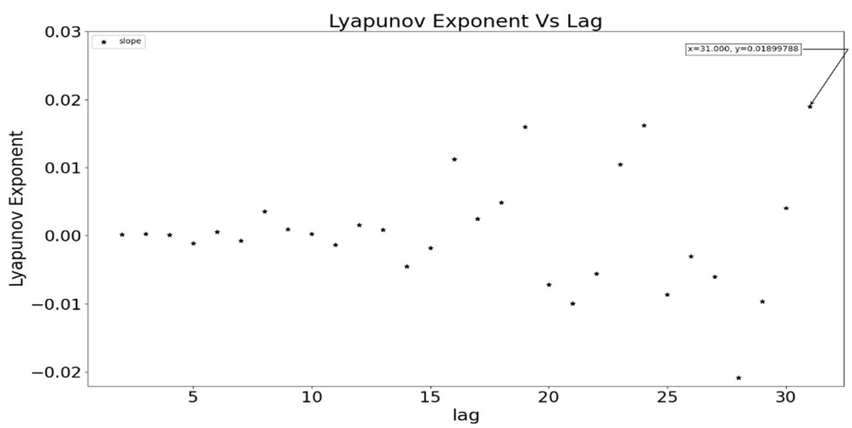


(Figure-33)

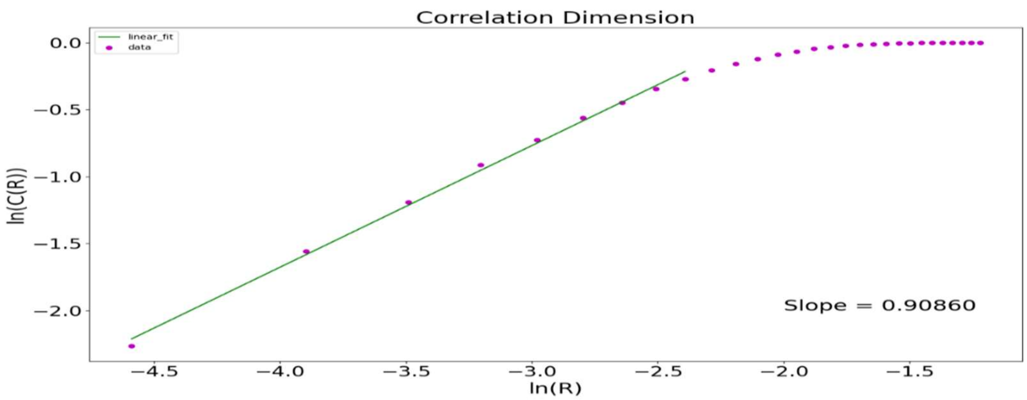
As Lyapunov exponent depends on initial value x_i, so thirty plots have been drawn,figure-3 to 32.[2] The slopes will give the Lyapunov exponents. Again, all the thirty Lyapunov exponents have been plotted with respect to their corresponding lags, figure -33. Plot shows positive and negative values of Lyapunov exponents, so average value has been calculated as, $\boldsymbol{\lambda = \sum_{i=1}^{30} \lambda(x_i)\ /30}$**=0.000397**, which is a positive value and indicates chaotic behavior of time series data.[6,7]

3.2Plot for Calculation of Correlation Dimension, $D_C(R)$

With the help of the equations,

$\boldsymbol{C}(\boldsymbol{R}) = \frac{\mathbf{1}}{\boldsymbol{N}(\boldsymbol{N}-\mathbf{1})}\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{N}}\sum_{\boldsymbol{i}\neq\boldsymbol{j},\boldsymbol{j}=\mathbf{1}}^{\boldsymbol{N}} \boldsymbol{\Theta}(\boldsymbol{R} - |\boldsymbol{x}_{\boldsymbol{i}} - \boldsymbol{x}_{\boldsymbol{j}}|)$ and $D_C = \lim_{R\to 0}\frac{\log C(R)}{\log R}$,

Plot of $ln\ C(R) \sim \ln R$ has been drawn, figure-34. The slope of plot, which is **0.90860**, the correlation dimension $D_C(R)$ of the attractor.A non-integral value of correlation dimension again indicates the presence of chaos.[2,3]



(Figure-34)

Remark: For figure-1 and figure-2, ORIGIN has been used for plotting. For figure-3 to figure-32 and figure-34, PYTHON 3.7 has been used for curve fitting and plotting the graphs. For plotting, matplotlib library has been used and linear regression has been performed using ordinary least square methods. Figure-33 is a scattering plot which has been drawn using PYTHON 3.7.

4. Result and Conclusion

Average value of Lyapunov exponent obtained by taking mean of slopes of plots shown in figure-3 to figure-32 is **0.000397**.Correlation dimension obtained as slope of plot shown in figure-34is **0.90860**.Positive value of Lyapunov exponent and no integral value of correlation dimension indicate the presence of chaos in the time series data of atmospheric temperature and hence in climate.[2][4][5][6] [7][8] The calculation of Lyapunov exponent and correlation dimension of time series data of other climate parameters would help in predicting the future behavior of climate.

5. Discussion and Future Work Plan

Stochastic methods are also used to forecast future trend in time series data of a climate parameter. Since climate is a complex system and shows chaotic behavior[11], so chaos theory is used to forecast the future behavior of climate parameters[5,8,9]. The formal use of chaos has been started since 1975 by Li and Yorke [3]. Today, chaos is widely studied and the concepts of quantifiers of chaos have been introduced to observe chaotic behavior in time series data. [2,3]In future, we will calculate minimum embedding dimension using Cao method [10] for the time series data of daily mean atmospheric temperature of a region in India. Calculation of minimum embedding dimension would help in deciding the minimum number of factors, which influence the atmospheric temperature. [8,9]

Acknowledgement

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