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METHODOLOGY FOR STUDYING A WAVE PACKET FOR FREE MOVEMENT IN QUANTUM MECHANICS USING A SEQUENCE OF A CAMPUS PROGRAM

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Annotation. Basic concepts of quantum mechanics. If the phenomena that occur in atoms are analyzed based on the laws of classical mechanics and electrodynamics, then experimental contradictory theoretical results can be obtained. The deep contradiction between theory and experimental results in this case requires fundamental changes in the laws and imaginations of classical physics to be made in the analysis of phenomena that occur in the presence of atoms. In this article, we will study some new principles of quantum mechanics using examples. In This, L.D.Landau and E.M.We work based on the manual of Lifshits.

Keyword: wave packet, quasiclassic boundary, maple system, uncertainty principle, integral exponential, classical approximation, abstract representation, wave function, wave packet probability flow density, Planck constant.

In the study of the wave packet for free movement, it should be noted that in the transition from quantum mechanics to classical mechanics, the motion described using the wave function does not move to the one that proceeds precisely on the trajectory. This requires the use of a special-view wavefunction (e.g., a wavepacket) to obtain a motion that runs along a specific trajectory. Moving at the quasiclassic boundary is necessary to change the dimensions of the sphere in conjunction with the Planck constant. In this case at the quasiclassic boundary, the wave packet can move along the classical trajectory of the particle in space.

Let's choose the most general view of a one-dimensional wave packet as follows

$$\psi(x,t) := \int_{-\infty}^{\infty} \psi(k,x,t)\, dk$$

here $\psi(k,x,t) := C(k)\, e^{(I(kx - wt))}$ va $w := \dfrac{h\,k^2}{2m}$.

In this case, for the approximation of the integral $C(k)$ amplitude $k\rangle\infty$ in condition, at least *1/k* must legally strive for zero. The arbitrary amplitude so chosen is clearly visible, leading to the Gai solution [1,2].

We choose to build a wave package like this, $t = 0$ the ability to find the particle in which the package is described in the starting time moment $p_0 = hq$ involuntary particle capable of moving $x = 0$ let the area around the point (with uncertainty ignoring the principle) be significantly different from zero. From these conditions, in particular,

$$\psi(x, 0) = A\, e^{\left(-\frac{x^2}{2\,a^2} + I q x \right)}$$

Spreading this function in series with respect to flat waves is expressed in the following way

$$\psi(x, 0) = \int C(k)\, e^{(I k x)}\, dk \quad ,$$

here $C(k) = \dfrac{A a}{\sqrt{2\pi}} \exp\left[-\dfrac{(k-q)^2}{2} \right]$.

Initial $t = 0$ with respect to the moment of time $C(k)$ by calculating the coefficient, it is possible to construct a wave packet at an arbitrary wattage moment, that is

$$\psi(x, t) = \frac{A a}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\left(-\frac{a^2 (k-q)^2}{2} + I k x - \frac{1}{2}\frac{I h k^2 t}{m} \right)}\, dk \quad .$$

We calculate this integral and determine its nature with respect to time[1,2]. We will try to carry out future calculations using the Maple system. We give values to the Integral subscript exponential argument [3]:

> **restart:**
arg:=-a^2/2*(k-q)^2+I*k*x-I*h*k^2/(2*m)*t;

$$arg := -\frac{1}{2} a^2 (k-q)^2 + I k x + \frac{-\frac{1}{2} I h k^2 t}{m}$$

To act on account of this integralna

> **otv:=int(exp(arg),k=-infinity..infinity);**

$$otv := \begin{cases} \dfrac{e^{\left(\dfrac{\frac{1}{2}I(-a^2 q^2 h t + 2 m a^2 q x + I m x^2)}{a^2 m + I h t}\right)} \sqrt{2}\sqrt{\pi}}{\sqrt{\dfrac{a^2 m + I h t}{m}}} & \text{csgn}\left(\dfrac{1}{2}a^2 + \dfrac{\frac{1}{2}I h t}{m}\right) = 1 \\ \infty & \text{otherwise} \end{cases}$$

The answer in this case gives the following function: if $c\,\text{sgn}\left(\dfrac{1\,a^2}{2} + \dfrac{1\,l\,h\,t}{2\,m}\right) = 1$, then the integral is equal to the exponent, in all other cases the integral moves away, i.e. equal to"infinite". An adequate response can be obtained from such a complex expression. Naturally, this requires knowing the type of expression obtained.

> **whattype(otv);**

$$function$$

For such a type of expression, it is possible to determine the number of xads in the expression and their sequence

> **nops(otv);**
op(otv);

$$3$$

$$\text{csgn}\left(\dfrac{1}{2}a^2 + \dfrac{\frac{1}{2}I h t}{m}\right) = 1, \quad \dfrac{e^{\left(\dfrac{\frac{1}{2}I(-a^2 q^2 h t + 2 m a^2 q x + I m x^2)}{a^2 m + I h t}\right)} \sqrt{2}\sqrt{\pi}}{\sqrt{\dfrac{a^2 m + I h t}{m}}}, \quad \infty$$

Thus, the necessary answer is –

> **op(2,otv)**

the type is the second operand in the function expression

$$\frac{e^{\left(\frac{\frac{1}{2}I(-a^2q^2ht+2ma^2qx+Imx^2)}{a^2m+Iht}\right)}\sqrt{2}\sqrt{\pi}}{\sqrt{\frac{a^2m+Iht}{m}}}$$

We gain practical(computations) experience with respect to the system of symbolic computations. Let's start by simplifying the calculation, First, the argument of the exponent located under the integral

> **factor(arg);**

$$-\frac{1}{2}\frac{a^2mk^2-2a^2mkq+a^2mq^2-2Ikxm+Ihk^2t}{m}$$

Direct computing does not give the necessary answer and, therefore, we use a series of systematized procedures of the student library (package

> **arg:=student[completesquare](arg,k);**

$$arg:=-\frac{1}{2}\frac{(a^2m+Iht)\left(k-\frac{m(a^2q+Ix)}{a^2m+Iht}\right)^2}{m}+\frac{\frac{1}{2}(-Ia^2q^2ht+2Ima^2qx-mx^2)}{a^2m+Iht}$$

The Argument breaks down into two adjuncts. One of them k is quadratic to its full vector, while the second is k is not bound to. Let's see how the Maple system works with such integrals. Originally of argnument k let's start work with a square-bound Part[4].

> **In:=int(exp(-b^2*z^2),z=-infinity..infinity);**

$$In := \begin{cases} \dfrac{\text{csgn}(b)\sqrt{\pi}}{b} & \text{csgn}(b^2) = 1 \\ \\ \infty & \textit{otherwise} \end{cases}$$

Encore b the parameter is chosen in relation to the work of the square, we get the rank[5]. However, $0 \langle b$ in additional terms

> **interface(showassumed=0);**
assume(b>0);
In:=unapply(int(exp(-b^2*z^2),z=-infinity..infinity),b);

$$In := b \rightarrow \frac{\sqrt{\pi}}{b\sim}$$

this integral easy solution kilindi

> **In(c);**

$$\frac{\sqrt{\pi}}{c}$$

We note that in this case we gave the tabular integral in the form of an image.

Thus, it remains to replace the integrable variables to calculate the integral. Thus, we extract the square terms in the argument

> **arg1:=op(1,arg);**

$$arg1 := -\frac{1}{2} \frac{(a^2 m + I h t)\left(k - \dfrac{m(a^2 q + I x)}{a^2 m + I h t}\right)^2}{m}$$

and let's introduce new variables

> **k0:=solve(arg1,k);**

$$k0 := \frac{m\,(a^2\,q + I\,x)}{a^2\,m + I\,h\,t},\ \frac{m\,(a^2\,q + I\,x)}{a^2\,m + I\,h\,t}$$

It is then possible to replace variables using the systematizable sense of the student packet[6].

> **In1:=student[changevar](k-k0[1]=y,Int(exp(arg1),k=-infinity..infinity),y);**

$$In1 := \int_{-\infty}^{\infty} e^{\left(-\frac{1}{2}\frac{(a^2\,m + I\,h\,t)\,y^2}{m}\right)}\, dy$$

It seems that not only the integral subspace representation has changed, but also the limits of integralization. Regardless, this integral can now be calculated by placing additional constraints on the values of the parameters[1,2].

In a different way, it is possible to divide the coefficients, to carry out the calculation with

> **assume(a>0,h>0,t>0,m>0);**
b0:=normal(sqrt(arg1/(k-k0[1])^2));

$$b0 := \frac{1}{2}\sqrt{-\frac{2\,a^2\,m + 2\,I\,h\,t}{m}}$$

and if we put them in a tabular integral, then

> **In(b0);**

$$2\,\frac{\sqrt{\pi}}{\sqrt{-\frac{2\,a^2\,m + 2\,I\,h\,t}{m}}}$$

Let's look at the part of the recent debate that does not depend on the size

> **arg2:=op(2,arg);**

$$arg2 := \frac{1}{2}\frac{-I a^2 q^2 h t + 2 I m a^2 q x - m x^2}{a^2 m + I h t}$$

x let's divide the part that is transparent to the variable

> **arg2:=student[completesquare](arg2,x);**

$$arg2 := -\frac{1}{2}\frac{m\left(x - I a^2 q\right)^2}{a^2 m + I h t} - \frac{1}{2}a^2 q^2$$

Thus, the result is

> **psi:=A*a/sqrt(2*Pi)*In(b0)*exp(arg2):**
psi:=normal(psi);

$$\psi := \frac{A a \sqrt{2}\, e^{\left(\frac{1}{2}\frac{-I a^2 q^2 h t + 2 I m a^2 q x - m x^2}{a^2 m + I h t}\right)}}{\sqrt{-\frac{2 a^2 m + 2 I h t}{m}}}$$

After computing the set function $\rho(x,t) = |\psi(x,t)|^2$ we see the behavior of the wave packet density itself

> **rho:=evalc(simplify(abs(psi)^2));**

$$\rho := \frac{a^2 m\, e^{\left(-\frac{a^2 m^2 x^2}{a^4 m^2 + h^2 t^2} + \frac{h t\left(-a^2 q^2 h t + 2 m a^2 q x\right)}{a^4 m^2 + h^2 t^2}\right)} A^2}{\sqrt{a^4 m^2 + h^2 t^2}}$$

We recommend, in this case, from the Maple system $evalc$ it is required to perform complex calculations using the command. ρ the exponential argument in a probability density expression consists of a sufficiently complex expression, and it is necessary to soddenize it

> **rho:=simplify(rho);**

$$\rho := \frac{a^2\, m\, e^{\left(-\frac{a^2\,(m\,x-q\,h\,t)^2}{a^4\,m^2+h^2\,t^2}\right)}\, A^2}{\sqrt{a^4\,m^2+h^2\,t^2}}$$

A comprehensive detail of the result obtained can be found in the literature[1,2].

Now we see the probability flow density of the wave packet

$$s(x,t)=\frac{h}{2\,m\,I}\left[\overline{(\Psi)}\left(\frac{\partial}{\partial x}\Psi\right)-\Psi\left(\frac{\partial}{\partial x}\overline{(\Psi)}\right)\right]$$

.

We do this in an abstract review in the Maple system to determine

> **s:=phi->(h/(2*I*m))*(conjugate(phi)*diff(phi,x)-phi*diff(conjugate(phi),x));**

$$s := \phi \rightarrow \frac{-\frac{1}{2}I\,h\left(\overline{(\phi)}\left(\frac{\partial}{\partial x}\phi\right)-\phi\left(\frac{\partial}{\partial x}\overline{(\phi)}\right)\right)}{m}$$

To this determination we can calculate the density by putting the wave function of the wave packet[3].

However, if the wave packet being viewed is therefore anilated, then the derivative of the wave functions can be defined

> **assume(q,real); assume(x,real);**

> **simplify(diff(psi,x)/psi);**

$$\frac{m\,(I\,a^2\,q-x)}{a^2\,m+I\,h\,t}$$

> **simplify(diff(conjugate(psi),x)/conjugate(psi));**

$$\frac{m\left(I a^2 q + x\right)}{-a^2 m + I h t}$$

without it, the properties of complete functions will be proportional to themselves. This achievement $s = v\rho$ denotes proportionality to magnitude, and v the coefficient takes the following view

> **v:=simplify(s(psi)/rho);**

$$v := -\frac{h\left|A\right|^2 \left(a^4 q m + x h t\right)}{A^2 \left(a^2 m + I h t\right)\left(-a^2 m + I h t\right)}$$

We note, q and x it is practically impossible without additional boundary conditions obtained with respect to variables. In this sense, we force the system to simplify the given expression

> **v:=simplify(evalc(v));**

$$v := \frac{h\left(a^4 q m + x h t\right)}{a^4 m^2 + h^2 t^2}$$

To do this, to describe the velocity dispersion in a graph, the private values of the parameters in the problem are v for size, it is necessary to put in the recorded expression

> **v1:=simplify(subs(h=1,m=1,a=1,q=2,v));**

$$v1 := \frac{2 + x t}{1 + t^2}$$

Then we describe the ratio of the probability flow density to the full probability, that is, the dependence on the coordinate at different moments of " speed

> **p0:=plot(subs(t=0,v1),x=-5..10,color=black,**
title='Dispersiyaning chiziqliligi', legend=`t=0`):
p1:=plot(subs(t=1,v1),x=-5..10,color=red,legend=`t=1`):
p2:=plot(subs(t=2,v1),x=-5..10,color=blue, legend=`t=2`):
p3:=plot(subs(t=3,v1),x=-5..10,color=green, legend=`t=3`):
plots[display](p0,p1,p2,p3);
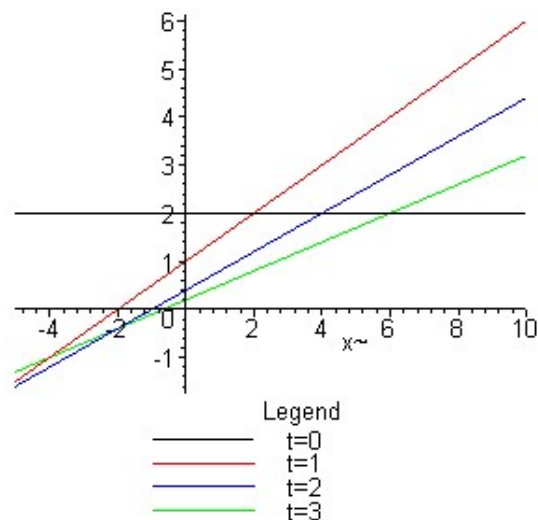
These straight lines describe the dispersion of velocity. Let's see the time-dependent variation (animation) of the velocity distribution in the gal

> **plots[animate](v1,x=-5..10,t=0..5,frames=30,color=blue,title='Izmenenie skorosto vo vremeni')[6];**
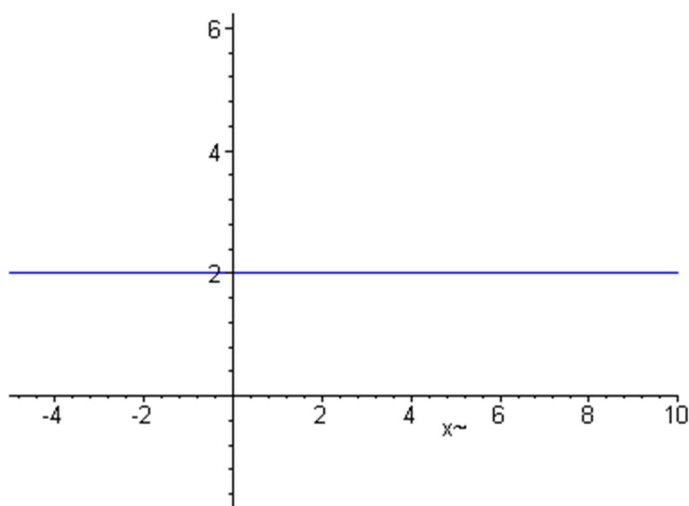
A sufficiently complex change in speed is characterized by the fact that different parts (founders) of the waveguide package move at different speeds, since in some cases, certain parts of the wave package can propagate even at negative speeds, in contrast to the wave propagation[7].
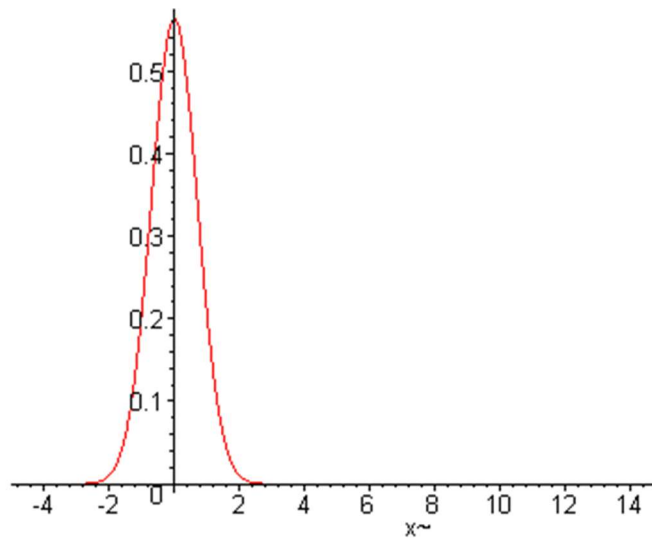
$\rho(x,t)$ we put the private values of the parameters in the density expression

> **rhoS:=simplify(subs(A=(Pi)^(-1/4),h=1,m=1,a=1,q=2,rho));**

$$rhoS := \frac{e^{\left(-\frac{(-x+2\,t)^2}{1+t^2}\right)}}{\sqrt{1+t^2}\,\sqrt{\pi}}$$

Let's see the change in density distribution with respect to icoordinate (animation

> **plots[animate](rhoS,x=-5..15,t=0..5,frames=30,color=red,numpoints=200,title='Izmenenie raspredeleniya plotnosti vo vremeni');**
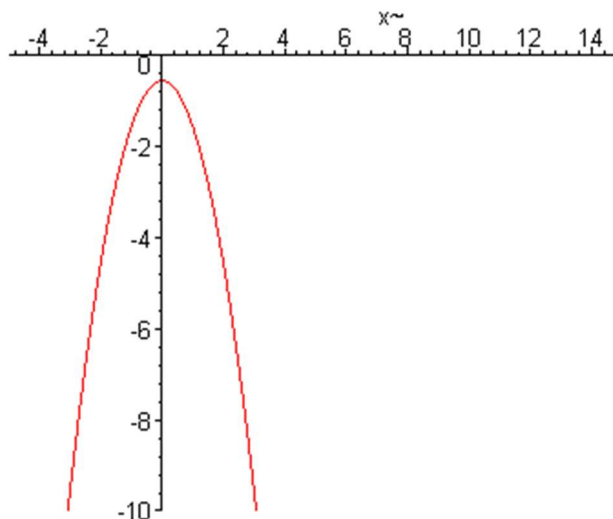

Full reverence, in this case, is invariable and equal to one

> **int(rhoS,x=-infinity..infinity)[5];**

$$1$$

The process of" watering " of the full package in our eyes $\log(\rho)$ density logarithm is described close to the so-called graph (animation).

> **LogRho:=simplify(log(rhoS)):**
plots[animate](LogRho,x=-5..15,t=0..5,view=[-5..15,-10..0],frames=30,color=red,numpoints=200,title='Izmenenie logarifma plotnosti vo vremeni');

Propagation of different parts of a wave packet at different speeds (even with respect to directions) leads to an increase in the particle coordinate error[5].

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