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GEOMETRIC WAY OF SOLVING PROBLEMS RELATED TO COMPUTATION WITH INVERSE TRIGONOMETRIC FUNCTIONS

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Annotation. This article provides information on the use of geometric methods in solving equations, when performing computations related to some inverse trigonometric functions.

Keywords: geometric method, inverse trigonometric functions, right-angled triangle, angle, function area of definition, function area of values, sines, and Pythagorean theorems.

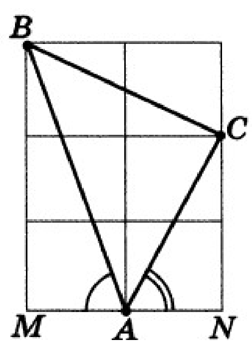
The geometric solution of algebraic problems is of great importance in increasing the interest of students in mathematics, in the cultivation of Mathematical Thinking, in the manifestation of the inextricable relationship between algebra and geometry. One of the necessary requirements is to learn it, to have knowledge about it, to be able to tassavur, to understand and apply it in essence, to study its features and to develop its methodology.

We will dwell in this article on geometric methods for adding, subtracting, solving equations related to Inverse trigonometric functions.

Example 1. $arctg1 + arctg2 + arctg3$ calculate.

Solution. According to drawing 1 $arctg3 = \angle BAM$, $arctg2 = \angle CAN$, $arctg1 = \angle BAC$. (BAC acute angle of a right-angled equilateral triangle).

So, $arctg1 + arctg2 + arctg3 = \pi$.



Draw 1

Answer: π

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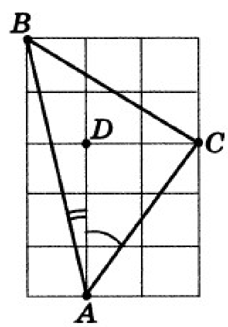
Example 2. $arctg\frac{2}{3}+arctg5$ calculate.

Solution. According to drawing 2 this gathering $\frac{\pi}{4}$ is equivalent to because

$$arctg\frac{2}{3}=\angle CAD,\ \ arctg5=\angle BAD\ ,$$

$\angle BAC$ right-angled equilateral ABC acute angle of the Triangle.



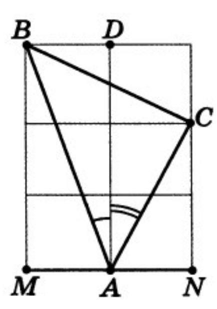
Draw 2

Answer: $\frac{\pi}{4}$.

Example 3. $\cos(arctg3+arctg0{,}5)$ calculate.

Solution. In drawing 3 ABC the Triangle is made of. In this $ctg\angle DAB=3$ and $tg\angle DAC=0{,}5$. ACB is a right-angled equilateral triangle. So,

$$arcctg3+arcctg0{,}5=\frac{\pi}{4}.\ \cos(arcctg3+arcctg0{,}5)=\frac{\sqrt{2}}{2}$$



Draw 3

Answer: $\frac{\sqrt{2}}{2}$.

Example 4. $tg\left(\arcsin\frac{2}{\sqrt{5}}+\arccos\frac{1}{\sqrt{10}}\right)$ calculate.

Solution. $\frac{2}{\sqrt{5}}>0$ since $\arcsin\frac{2}{\sqrt{5}}$ the angle of the right triangle, whose catheters are relatively 1:2. Then this is the magnitude of the angle $arctg2$ can be viewed as. Similarly, by analyzing $\arccos\frac{1}{\sqrt{10}}=arctg3$ we form the. According to drawing 3 $\angle MAB=arctg3$ and $\angle NAC=arctg2$. Their sum is $\pi-\frac{\pi}{4}$ is equivalent to. So, $tg\left(arctg\frac{2}{\sqrt{5}}+\arccos\frac{1}{\sqrt{10}}\right)=tg\left(\pi-\frac{\pi}{4}\right)=-1$.

Answer: -1.

Now let's consider the solution of the following general issue.

Example 5. $f(x)=arctg1+arctg\left(1+\frac{1}{x}\right)+arctg(1+2x)$ find the value domain of the function.

Solution. The domain of definition of this function is all real numbers other than zero

$$D(f)=(-\infty;0)\cup(0;\infty)$$

$f(x)$ value domain of the function $E(f)$ let's consider 5 cases of finding.

1) $\begin{cases}1+\frac{1}{x}<0\\ 1+2x<0\end{cases}$

In this case $-1<x<-\frac{1}{2}$ and $A(x)<0$, in this

$$A(x)=arctg\left(1+\frac{1}{x}\right)+arctg(1+2x),\quad tg\,A(x)=\frac{1+\frac{1}{x}+1+2x}{1-\left(1+\frac{1}{x}\right)(1+2x)}=\frac{2+2x+\frac{1}{x}}{-2-2x-\frac{1}{x}}=-1.$$

$2+2x+\frac{1}{x}<0$ since $\left(-1;-\frac{1}{2}\right)$ all in the range x for

$$A(x) = -\frac{\pi}{4}; \quad f(x) = arctg1 + A(x) = \frac{\pi}{4} - \frac{\pi}{4} = 0$$

2) $\begin{cases} 1 + \dfrac{1}{x} < 0 \\ 1 + 2x > 0 \end{cases}$

In this case $-\frac{1}{2} < x < 0$ and $B(x) > 0$ in this $B(x) = arctg1 + arctg(1 + 2x)$.

$$tgB(x) = \frac{1 + 1 + 2x}{1 - 1 + (1 + 2x)} = \frac{2 + 2x}{-2x} = -\left(1 + \frac{1}{x}\right), \ B(x) = -arctg\left(1 + \frac{1}{x}\right).$$

So, $f(x) = arctg(1 + \frac{1}{x}) + B(x) = \mathrm{arctg}(1 + \frac{1}{x}) - arctg(1 + \frac{1}{x}) = 0$

3) $\begin{cases} 1 + \dfrac{1}{x} > 0 \\ 1 + 2x < 0 \end{cases}$

In this case $x < -1$ and $C(x) > 0$, in this $C(x) = arctg1 + arctg\left(1 + \frac{1}{x}\right)$

$$tgC(x) = \frac{1 + 1 + \dfrac{1}{x}}{1 - 1 \cdot (1 + \dfrac{1}{x})} = \frac{2 + \dfrac{1}{x}}{-\dfrac{1}{x}} = -(1 + 2x), \ tg\,C(x) = -arctg(1 + 2x).$$

So, $f(x) = C(x) + arctg(1 + 2x) = -arctg(1 + 2x) + arctg(1 + 2x) = 0$

4) $f(-1) = arctg1 + arctg\left(1 + \frac{1}{-1}\right) + arctg(1 + 2(-1)) = 0$.

$$f\left(-\frac{1}{2}\right) = arctg1 + arctg\left(1 + \frac{1}{-\dfrac{1}{2}}\right) + arctg\left(1 + 2 \cdot \left(-\frac{1}{2}\right)\right) = 0$$

So, $x < 0$ in $f(x) = 0$.

5) $\begin{cases} 1+\dfrac{1}{x}>0 \\ 1+2x>0 \end{cases}$ $\qquad D(x)=arctg\left(1+\dfrac{1}{x}\right)+arctg(1+2x)$,

$$tgD(x)=\frac{1+\frac{1}{x}+1+2x}{1-\left(1+\frac{1}{x}\right)(1+2x)}=\frac{2+2x+\frac{1}{x}}{-2-2x-\frac{1}{x}}=-1.$$

$(0;+\infty)$ in the range x for all values of

$$2+2x+\frac{1}{x}>0\,, D(x)=\frac{3\pi}{4};\quad f(x)=arctg1+D(x)=\frac{\pi}{4}+\frac{3\pi}{4}=\pi\ .$$

So, $x>0$ da $f(x)=\pi$.

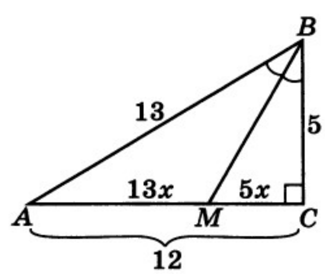
Now it is not difficult to find the area of values of a given function.

$$E(f)=\{0;\pi\}$$

Example 6. $ctg\left(\dfrac{1}{2}\arccos\dfrac{5}{13}\right)$ calculate.

Solution. An example is instantaneous if we apply the concept of cosine and catangency of the acute angle of a right triangle, Pythagorean theorem, and the property of the bisector.

In drawing 4 $\angle ACB=90^0$ is ABC the Triangle is depicted. $BC=5$, $AB=13$ and BM ABC angle bisector.



Draw 4

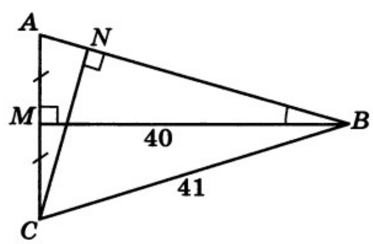
Then $MC=5x, AM=13x,\ AC=12,\ x=\dfrac{2}{3}$.

$$ctg\left(\frac{1}{2}\arccos\frac{5}{13}\right)=\frac{BC}{MC}=\frac{5}{5x}=\frac{1}{x}=\frac{3}{2}.$$

Answer: $\frac{3}{2}$.

Example 7. $\sin(2\arccos\frac{40}{41})$ calculate.

Solution. In drawing 5 ABC an equilateral triangle is described. $(AB=BC=41)$ $BM\perp AC$, $CN\perp AB$.



. Draw 5

By Pythagorean theorem $AM=9$. Then

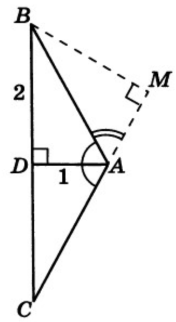
$$\sin\left(2\arccos\frac{40}{41}\right)=\frac{CN}{BC}, BC=41, CN=\frac{AC\cdot BM}{AB}, \ CN=\frac{2\cdot 9\cdot 40}{41}=\frac{720}{41}.$$

So, $\sin\left(2\arccos\frac{40}{41}\right)=-\frac{720}{1681}$.

Answer: $\frac{720}{1681}$.

Example 8. $\cos(2arctg\,2)$ calculate.

Solution. Triangle according to drawing 6 BAC in $\angle BAC=2arctg\,2$



Draw 6

Because this angle does not pass $arctg 2 > arctg 1$

So, $\cos(2arctg 2) = -\cos\angle BAM$.

$$\cos\angle BAM = \frac{AM}{AB}.$$

By Pythagorean theorem, ΔBAM for $AM = \sqrt{AB^2 - BM^2}$. ΔABD for $AB = \sqrt{5}$

BM in ABC as the height of the triangle we find.

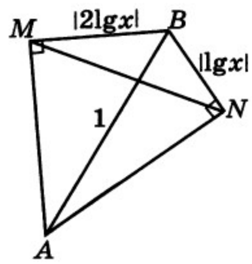
$$BM = \frac{2S_{\Delta ABC}}{AC}.$$

So, $BM = \frac{4}{\sqrt{5}}$. AM calculating, $\cos(2arctg 2) = -\frac{3}{5}$ we find.

Answer: $\cos(2arctg 2) = -\frac{3}{5}$

Example 9. This $\arcsin \lg x^2 + \arcsin \lg x = \frac{\pi}{3}$ solve the equation.

Solution. Hypotenuse $AB = 1$ is ABM and ABN we look at the triangles.(Draw 7 $MB = |2\lg x|$ va $NB = |\lg x|$ let. Then $\angle MAN$ according to the condition $\frac{\pi}{3}$ will have a size. A, M, B, N diameter AB lies in the circle that is. Then MN by the vatar length sine theorem $AB \sin\frac{\pi}{3}$ is equivalent to. $MN = \frac{\sqrt{3}}{2}$.



Draw 7

On the other hand MBN according to the cosines theorem in a triangle.

$$MN^2 = MB^2 + BN^2 - 2 \cdot MB \cdot BN \cos \angle MBN$$

$$\cos \angle MBN = \cos \frac{2\pi}{3} = -\frac{1}{2}$$

then

$$(\frac{\sqrt{3}}{2})^2 = (2\lg x)^2 + (\lg x)^2 - 2 \cdot 2\lg x \cdot \lg x \cdot \left(-\frac{1}{2}\right)$$

and

$$7\lg^2 x = \frac{3}{4},\ |\lg x| = \frac{\sqrt{221}}{14}$$

from this

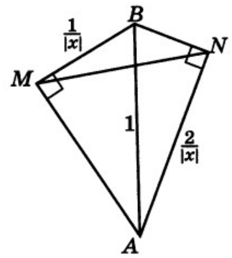
$$x_1 = 0{,}1^{\frac{\sqrt{21}}{14}}; \qquad x_2 = 10^{\frac{\sqrt{21}}{14}}.$$

Both values $|\lg x| \le \frac{1}{2}$ satisfies the condition. But x_1 as a result of verification, the equation will not have a root.

Answer: $10^{\frac{\sqrt{21}}{14}}$.

Example 10. $\arcsin\frac{1}{x} + \arccos\frac{2}{x} = \frac{\pi}{6}$ calculate.

Solution. As you know, $|x| \ge 2$. Hypotenuse $AB = 1$ is ABM and ABN we look at the right triangles. (Draw 8).



Draw 8

$BM = \frac{1}{|x|}$ and $AN = \frac{2}{|x|}$ let. Then $\angle MAN = \frac{\pi}{6}$. A, M, B, N the dots have a diameter of. AB lies in the circle that is. Then MN by the vatar length sine theorem $AB\sin\frac{\pi}{6}$ is equivalent to. $MN = \frac{1}{2}$.

$$MN^2 = AM^2 + AN^2 - 2 \cdot AM \cdot AN \cos\angle MAN$$

$$\frac{1}{4} = 1 - \frac{1}{x^2} + \frac{4}{x^2} - 2\sqrt{1 - \frac{1}{x^2}} \cdot \frac{2}{|x|} \cdot \frac{\sqrt{3}}{2}, \quad \frac{3}{x^2} + \frac{3}{4} = 2\sqrt{3}\sqrt{\frac{1}{x^2} - \frac{1}{x^4}}$$

To make it easier for you to solve $\frac{1}{x^2}$ in t substituting, $112t^2 - 40t + 3 = 0$ we form the. In this $t_1 = \frac{3}{28}; t_2 = \frac{1}{4}$. Back to marking $x = 2$ we determine that the given equation is the root.

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