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STATEMENT AND INVESTIGATION OF A BOUNDARY VALUE PROBLEM FOR A THIRD-ORDER PARABOLIC-HYPERBOLIC EQUATION OF THE FORM $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+c\right)(Lu)=0$ IN A CONCAVE HEXAGONAL AREA WITH TWO LINES FOR CHANGING THE TYPE

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Annotation. In this paper, we present and investigate a boundary value problem for a third-order parabolic-hyperbolic equation in the form $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+c\right)(Lu)=0$ of a concave hexagonal sphere with two types of exchange lines.

Keywords: parabolic-hyperbolic type, boundary value problem, line of type change, solution of an equation, integral equation, differential equation, concave hexagonal domain.

Introduction

Since the 70-80 years of the XX century, the study of various boundary value problems for third-and high-order equations of parabolic-hyperbolic type has been started. Such problems were mainly studied by T. D. Juraevand his students (for example, see [1], [2]).

At present, the study of boundary value problems for third-and high-order equations is being developed broadlum the plan (for example, see [3] - [7] and others).

Problem statement

In the domain G of the plane xOy, consider the equation

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+c\right)(Lu)=0 \tag{1}$$

where $c\in R$, $Lu\equiv\begin{cases}L_1u\equiv u_{xx}-u_y & (x,y)\in G_1,\\ L_iu\equiv u_{xx}-u_{yy} & (x,y)\in G_i\ (i=2,3),\end{cases}$ $G=G_1\cup G_2\cup G_3\cup J_1\cup J_2$, G_1- is a polygonwith vertices at points $A(0,\,0)$, $B(1,\,0)$, $B_0(1,\,1)$, $A_0(0,\,1)$; G_2- is a triangle with vertices at points $A(0,\,0)$, $B(1,\,0)$, $C(0,-1)$; G_3- is a rectangle with vertices at points $A(0,\,0)$, $A_0(0,\,1)$, $D_0(-1,\,1)$, $D(-1,0)$; J_1- is an open segment with vertices at points $A(0,\,0)$and $B(1,\,0)$; J_2- is an open segment with vertices at points $A(0,\,0)$and $A_0(0,\,1)$, i.e. $G-$ is a concave hexagonal region with vertices at points $A(0,\,0)$, $C(0,-1)$, $B(1,\,0)$, $B_0(1,\,1)$, $D_0(-1,\,1)$, $D(-1,0)$.

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Area G_2 Use a line segment to divide the area AE into two parts. Then this area can be written as: $G_2 = G_{21} \cup G_{22} \cup AE$, where $G_{21} -$ is a triangle with vertices at points $A(0,\ 0)$, $B(1,\ 0)$, $E(1/2,\ -1/2)$; $G_{22} -$ is a triangle with vertices at points $A(0,\ 0)$, $C(0,-1)$, $E(1/2,\ -1/2)$; $AE -$ is an open segment with vertices at points $A(0,\ 0)$and $E(1/2,\ -1/2)$.

Now we proceed to the formulation of the following boundary value problem for equation (1):

A task M_{11c}**.** Find a function $u(x,y)$, that: 1) is continuous in a closed domain $\overline{G}$; 2) in each of the domains $G_i\ (i=1,2,3)$ satisfies equation (1), and the derivatives u_x, u_y, u_{xx} and u_{yy} are continuous up to the part of the boundary specified in the boundary conditions; 3) satisfies the following boundary conditions:

$$u(1,y)=\varphi_1(y),\quad 0\le y\le 1, \qquad (2)$$

$$u(-1,y)=\varphi_2(y),\ 0\le y\le 1, \qquad (3)$$

$$u_x(-1,y)=\varphi_3(y),\ 0\le y\le 1, \qquad (4)$$

$$u(0,y)=\varphi_4(y),\ -1\le y\le 0, \qquad (5)$$

$$u_x(0,y)=\varphi_5(y),\ -1\le y\le 0, \qquad (6)$$

$$u_{xx}(0,y)=\varphi_6(y),\quad -1\le y\le 0, \qquad (7)$$

$$u(x,0)=f_1(x),\quad -1\le x\le 0, \qquad (8)$$

$$u_y(x,0)=f_2(x),\quad -1\le x\le 0, \qquad (9)$$

$$u_{yy}(x,0)=f_3(x),\quad -1\le x\le 0 \qquad (10)$$

and 4) satisfies continuous gluing conditions on lines of type change:

$$u(x,\ -0)=u(x,+0)=\tau_1(x),\ 0\le x\le 1, \qquad (11)$$

$$u_y(x,\ -0)=u_y(x,\ +0)=\nu_1(x),\ 0\le x\le 1, \qquad (12)$$

$$u_{yy}(x,\ -0)=u_{yy}(x,\ +0)=\mu_1(x),\ 0<x<1, \qquad (13)$$

$$u(-0,\ y)=u(+0,\ y)=\tau_2(y),\ 0\le y\le 1, \qquad (14)$$

$$u_x(-0,\ y)=u_x(+0,y)=\nu_2(y),\ 0\le y\le 1, \qquad (15)$$

$$u_{xx}(-0,y)=u_{xx}(+0,y)=\mu_2(y),\ 0<y<1, \qquad (16)$$

where $\varphi_i\ \left(i=\overline{1,6}\right)$, $f_k\ (k=1,2,3)$ – are given sufficiently smooth functions, $\tau_k,\nu_k,\mu_k\ (k=1,2)$ – are unknown yet sufficiently smooth functions, and the matching conditions $\tau_1(0)=\tau_2(0)$, $\nu_1(0)=\tau_2'(0)$, $\tau_1'(0)=\nu_2(0)$are satisfied .

Let us formulate the following theoremy:

The theorem. If $\varphi_1,\varphi_2\in C^3[0,1]$, $\varphi_3\in C^2[0,1]$, $\varphi_4\in C^3[-1,0]$, $\varphi_5\in C^2[-1,0]$, $\varphi_6\in C^1[-1,0]$, $f_1\in C^3[-1,0]$, $f_2\in C^2[-1,0]$, $f_3\in C^1[-1,0]$, and the matching conditions $f_1(-1)=\varphi_2(0)$are met , $f_1'(-1)=\varphi_3(0)$, $f_1(0)=\varphi_4(0)=\tau_1(0)=\tau_2(0)$, $f_2(-1)=\varphi_2'(0)$,

$f_1'(0)=\varphi_5(0)=\tau_1'(0)=\nu_2(0)$, $\quad f_2(0)=\varphi_4'(0)=\nu_1(0)=\tau_2'(0)$, $\quad f_3(0)=\varphi_4''(0)$, $\quad f_1''(0)=\varphi_6(0)$, $f_3(-1)=\varphi_2''(0)$, then the problem M_{11c} starts with a unique solution.

Proof. To provethe theorem, by introducing the notation $Lu=v$, we rewrite equation (1) in the form $\frac{\partial v}{\partial x}+\frac{\partial v}{\partial y}+cv=0$. The general solution of the last equation has the form $v=\omega(x-y)e^{-cy}$. Then we get

$$Lu_i=\omega_i(x-y)e^{-cy},$$

where the notation is introduced

$$u(x,y)=u_i(x,y),\omega(x-y)=\omega_i(x-y),\ (x,y)\in D_i\ (i=1,2,3) \tag{17}$$

The last equation can be written as

$$u_{1xx}-u_{1y}=\omega_1(x-y)e^{-cy}, \tag{18}$$

$$u_{ixx}-u_{iyy}=\omega_i(x-y)e^{-cy}\ (i=2,3), \tag{19}$$

where $\omega_i(x-y)\ (i=1,2,3)$ – the unknown functions are still sufficiently smooth functions.

If (19) $(i=2)$ we introduce the notation, in equation (19) $u_2(x,y)=u_{2k}(x,y)$, $\omega_2(x-y)=\omega_{2k}(x-y)\ \ ((x,y)\in D_{2k}\ (k=1,2))$, then equation (19) $(i=2)$ takes the form

$$u_{2kxx}-u_{2kyy}=\omega_{2k}(x-y)e^{-cy}\ (k=1,2). \tag{20}$$

First M_{11c} , we will investigate this problem in the following areas G_2: If we take into account the form of the domain G_2, then passing in equation (20) ($k=2$) to the limit for by $x\to 0$ virtue of (5) and (7), we find:

$$\omega_{22}(-y)=[\varphi_6(y)-\varphi_4''(y)]e^{cy}.$$

Here, changing $-y$ na $x-y$, we get

$$\omega_{22}(x-y)=[\varphi_6(y-x)-\varphi_4''(y-x)]e^{c(y-x)}.$$

Now we write down the solution of equation (20) ($k=1$), that satisfies conditions (11), (12):

$$u_{21}(x,y)=\frac{\tau_1(x+y)+\tau_1(x-y)}{2}+\frac{1}{2}\int_{x-y}^{x+y}\nu_1(t)dt-\frac{1}{2}\int_0^y e^{-c\eta}d\eta\int_{x-y+\eta}^{x+y-\eta}\omega_{21}(\xi-\eta)d\xi. \tag{21}$$

Differentiation dierentiating (21) with respect to x and y, we obtain:

$$u_{21x}(x,y)=\frac{\tau_1'(x+y)+\tau_1'(x-y)}{2}+\frac{1}{2}\left[\nu_1(x+y)-\nu_1(x-y)\right]-$$

$$-\frac{1}{2}\int_0^y\left[\omega_{21}(x+y-2\eta)-\omega_{21}(x-y)\right]e^{-c\eta}d\eta, \tag{22}$$

$$u_{21y}(x,y)=\frac{\tau_1'(x+y)-\tau_1'(x-y)}{2}+\frac{1}{2}\left[\nu_1(x+y)+\nu_1(x-y)\right]-$$

$$-\frac{1}{2}\int_0^y\left[\omega_{21}\left(x+y-2\eta\right)+\omega_{21}\left(x-y\right)\right]e^{-c\eta}d\eta\,. \tag{23}$$

As above, we write down the solution of equation (20) ($k=2$)that satisfies conditions (5) and (6):

$$u_{22}\left(x,y\right)=\frac{\varphi_4\left(y+x\right)+\varphi_4\left(y-x\right)}{2}+\frac{1}{2}\int_{y-x}^{y+x}\varphi_5\left(t\right)dt+\frac{1}{2}\int_0^x d\eta\int_{y-x+\eta}^{y+x-\eta}\omega_{21}\left(\eta-\xi\right)e^{-c\xi}d\xi\,. \tag{24}$$

Differentiation dierentiating (2-4) with respect to x and y , we obtain

$$u_{22x}\left(x,y\right)=\frac{\varphi_4'\left(y+x\right)-\varphi_4'\left(y-x\right)}{2}+\frac{1}{2}\left[\nu_3\left(y+x\right)+\nu_3\left(y-x\right)\right]+$$

$$+\frac{1}{2}\int_0^x\left[\omega_{22}\left(2\eta-x-y\right)e^{-c\left(y+x-\eta\right)}+\omega_{22}\left(x-y\right)e^{-c\left(y-x+\eta\right)}\right]d\eta\,, \tag{25}$$

$$u_{22y}\left(x,y\right)=\frac{\varphi_4'\left(y+x\right)+\varphi_4'\left(y-x\right)}{2}+\frac{1}{2}\left[\nu_3\left(y+x\right)-\nu_3\left(y-x\right)\right]+$$

$$+\frac{1}{2}\int_0^x\left[\omega_{22}\left(2\eta-x-y\right)e^{-c\left(y+x-\eta\right)}-\omega_{22}\left(x-y\right)e^{-c\left(y-x+\eta\right)}\right]d\eta\,. \tag{26}$$

Now substituting (22), (23), (25) and (26) in the condition

$$\left.\left(\frac{\partial u_{21}}{\partial x}+\frac{\partial u_{21}}{\partial y}\right)\right|_{y=-x}=\left.\left(\frac{\partial u_{22}}{\partial x}+\frac{\partial u_{22}}{\partial y}\right)\right|_{y=-x},$$

we get the equality

$$\tau_1'\left(0\right)+\nu_1\left(0\right)-\int_0^{-x}\omega_{21}\left(-2\eta\right)\exp\left(-c\eta\right)d\eta=\varphi_4'\left(0\right)+\nu_3\left(0\right)+\int_0^x\omega_{22}\left(2\eta\right)e^{c\eta}d\eta,\ \ 0\le x\le 1/2\,.$$

In order toifferentsiderive this equality, we find

$$\omega_{21}\left(2x\right)e^{cx}=\omega_{22}\left(2x\right)e^{cx},\ 0\le x\le 1/2$$

Reducing the last equality by e^{cx} and changing $2x$ to $x-y$, we have

$$\omega_{21}\left(x-y\right)=\omega_{22}\left(x-y\right),\ 0\le x-y\le 1\,.$$

Now substituting (21) and (24) into the condition $u_{21}(x,-x)=u_{22}(x,-x)$, we arrive at the equation

$$\frac{\tau_1\left(0\right)+\tau_1\left(2x\right)}{2}+\frac{1}{2}\int_{2x}^0\nu_1\left(t\right)dt-\frac{1}{2}\int_0^{-x}e^{-c\eta}d\eta\int_{2x+\eta}^{-\eta}\omega_{21}\left(\xi-\eta\right)d\xi=\psi_1\left(x\right),\ 0\le x\le 1/2\,,$$

where

$$\psi_1\left(x\right)=\frac{\varphi_4\left(0\right)+\varphi_4\left(-2x\right)}{2}+\frac{1}{2}\int_{-2x}^0\varphi_5\left(t\right)dt+\frac{1}{2}\int_0^x d\eta\int_{\eta-2x}^{-\eta}\omega_{21}\left(\eta-\xi\right)e^{-c\xi}d\xi\ -$$

a known function.

Deriving thelastequation, we obtain

$$\tau_1'(2x)-\nu_1(2x)==\psi_1(x)+\omega_{21}(2x)\int_0^{-x} e^{-c\eta}d\eta,\ 0\le x\le 1/2.$$

Here, changing $2x$ to x, we get the first relation between $\tau_1(x)$ and $\nu_1(x)$:

$$\tau_1'(x)-\nu_1(x)=\alpha_1(x),\ \ 0\le x\le 1, \tag{27}$$

where $\alpha_1(x)=\psi_1'\left(\frac{x}{2}\right)+\omega_{21}(x)\int_0^{-\frac{x}{2}} e^{-c\eta}d\eta$.

Now we rewrite equation (1) as

$$u_{1xxx}+u_{1xxy}+cu_{1xx}-u_{1xy}-u_{1yy}-cu_{1y}=0.$$

In this equation and in equation (20) ($k=1$) , passing to the limit at $y\to 0$ and taking into account the conditions (11), (12), (13), we get the equations

$$\tau_1'''(x)+\nu_1''(x)+\tau_1''(x)-\nu_1'(x)-\mu_1(x)-c\nu_1(x)=0,$$

$$\tau_1''(x)-\mu_1(x)=\omega_{21}(x).$$

By excluding from these equations and from equation (2-7) the functions $\nu_1(x)$ and $\mu_1(x)$, we arrive at the equation

$$\tau_1'''(x)-\left(1-\frac{c}{2}\right)\tau_1''(x)-\frac{c}{2}\tau_1'(x)=\frac{1}{2}\alpha_1''(x)-\frac{1}{2}\alpha_1'(x)-\frac{1}{2}\left[c\alpha_1(x)+\omega_{21}(x)\right].$$

Integrating this equation from 0 to x, we have

$$\tau_1''(x)-\left(1-\frac{c}{2}\right)\tau_1'(x)-\frac{c}{2}\tau_1(x)=\alpha_2(x)+k_1, \tag{28}$$

where

$$\alpha_2(x)=\frac{1}{2}\alpha_1'(x)-\frac{1}{2}\alpha_1(x)-\frac{1}{2}\int_0^x\left[c\alpha_1(t)+\omega_{21}(t)\right]dt-$$

known function, k_1 – a constant that is not yet known.

When solving equation (2-8), there can be three cases:: 1) $c\ne -2$, $c\ne 0$; 2) $c=-2$; 3) $c=0$.

Consider the case 1) ($c\ne -2$, $c\ne 0$). In this case, the characteristic equation of equation (28) has two different real roots: $\lambda_1=1, \lambda_2=-\frac{c}{2}$. We solve equation (28) under the conditions

$$\tau_1(0)=f_1(0),\ \tau_1'(0)=f_1'(0),\ \tau_1''(0)=f_1''(0). \tag{29}$$

First, we write down the general solution of the homogeneous equation corresponding to equation (2-8):

$$\tau_{10}(x)=C_1e^x+C_2e^{-\frac{c}{2}x}.$$

To find the general solution of equation (28), we use the method of variation of arbitrary constants, i.e. , we look for the general solution of equation (28) in the form

$$\tau_1(x) = C_1(x)e^x + C_2(x)e^{-\frac{c}{2}x}. \tag{30}$$

Differentiation let's differentiate (30):

$$\tau_1'(x) = C_1'(x)e^x + C_2'(x)e^{-\frac{c}{2}x} + C_1(x)e^x - \frac{c}{2}C_2(x)e^{-\frac{c}{2}x}.$$

Functions $C_1(x)$ и $C_2(x)$ We select the functions and so, that the equality is fulfilled

$$C_1'(x)e^x + C_2'(x)e^{-\frac{c}{2}x} = 0. \tag{31}$$

Now we find $\tau_1''(x)$:

$$\tau_1''(x) = C_1'(x)e^x - \frac{c}{2}C_2'(x)e^{-\frac{c}{2}x} + C_1(x)e^x + \frac{c^2}{4}C_2(x)e^{-\frac{c}{2}x}.$$

Functions $C_1(x)$ и $C_2(x)$ We select the functions and so, that the equality is fulfilled

$$C_1'(x)e^x - \frac{c}{2}C_2'(x)e^{-\frac{c}{2}x} = \alpha_2(x) + k_1. \tag{32}$$

Now solving the system (31) and (32), we find $C_1'(x)$ va $C_2'(x)$

$$C_1'(x) = \frac{2}{2+c}\left[\alpha_2(x) + k_1\right]e^{-x}, \quad C_2'(x) = -\frac{2}{2+c}\left[\alpha_2(x) + k_1\right]e^{\frac{c}{2}x}.$$

Integrating these equalities from 0 to x, we find

$$C_1(x) = \frac{2}{2+c}\int_0^x e^{-t}\alpha_2(t)dt - \frac{2k_1}{2+c}\left(e^{-x} - 1\right) + k_2,$$

$$C_2(x) = -\frac{2}{2+c}\int_0^x e^{\frac{c}{2}t}\alpha_2(t)dt - \frac{2k_1}{2+c}\cdot\frac{c}{2}\left(e^{\frac{c}{2}x} - 1\right) + k_3,$$

where k_2, k_3 are currently unknown constants.

Substituting the values of the functions $C_1(x)$ and $C_2(x)$ in (30), we find

$$\tau_1(x) = \frac{2}{2+c}\int_0^x \left[e^{x-t} - e^{\frac{c}{2}(t-x)}\right]\alpha_2(t)dt -$$

$$-\frac{2k_1}{2+c}\left[1 - e^x + \frac{2}{c}\left(1 - e^{-\frac{c}{2}x}\right)\right] + k_2e^x + k_3e^{-\frac{c}{2}x}. \tag{33}$$

Differentiating (33) twice sequentially, we obtain

$$\tau_1'(x) = \frac{2}{2+c}\int_0^x \left[e^{x-t} + \frac{c}{2}e^{\frac{c}{2}(t-x)}\right]\alpha_2(t)dt -$$

$$-\frac{2k_1}{2+c}\left(e^{-\frac{c}{2}x} - e^x\right) + k_2e^x - \frac{c}{2}k_3e^{-\frac{c}{2}x}, \tag{34}$$

$$\tau_1''(x) = \alpha_2(x) + \frac{2}{2+c}\int_0^x \left[e^{x-t} + \frac{c}{2} e^{\frac{c}{2}(t-x)} \right] \alpha_2(t)dt +$$

$$+ \frac{2k_1}{2+c}\left(\frac{c}{2} e^{-\frac{c}{2}x} + e^x \right) + k_2 e^x + \frac{c^2}{4} k_3 e^{-\frac{c}{2}x} . \qquad (35)$$

Now substituting (33), (34) and (35) into conditions (29), respectively, we find

$$k_3 = \frac{2}{2+c}\left[f_1(0) - f_1'(0) \right],\ k_2 = \frac{c}{2+c} f_1(0) + \frac{2}{2+c} f_1'(0),$$

$$k_1 = f_1''(0) - \alpha_2(0) - \left(k_2 + \frac{c^2}{4} k_3 \right).$$

Now consider case 2) ($c = -2$). In this case, equation (2-8) takes the form

$$\tau_1''(x) - 2\tau_1'(x) + \tau_1(x) = \alpha_2(x) + k_1 . \qquad (36)$$

The characteristic equation of this equation has one two-fold root $\lambda = 1$. In this case, the general solution of the homogeneous equation corresponding to equation (36) has the form

$$\tau_{10}(x) = (C_1 + C_2 x)e^x .$$

Then, as above, we will look for the general solution of equation (36) in the form

$$\tau_1(x) = \left[C_1(x) + xC_2(x) \right] e^x . \qquad (37)$$

We differentiate (37):

$$\tau_1'(x) = \left[C_1'(x) + xC_2'(x) \right] e^x + \left[C_1(x) + (x+1)C_2(x) \right] e^x .$$

Functions $C_1(x)$ и $C_2(x)$ We select the functions and so, that the equality is fulfilled

$$C_1'(x) + xC_2'(x) = 0 . \qquad (38)$$

Now we find $\tau_1''(x)$:

$$\tau_1''(x) = \left[C_1'(x) + (x+1)C_2'(x) \right] e^x + \left[C_1(x) + (x+2)C_2(x) \right] e^x .$$

Functions $C_1(x)$ и $C_2(x)$ We select the functions and so, that the equality is fulfilled

$$\left[C_1'(x) + (x+1)C_2'(x) \right] e^x = \alpha_2(x) + k_1 . \qquad (39)$$

From (38) and (39) we find $C_1'(x)$ and $C_2'(x)$:

$$C_1'(x) = -x\left[\alpha_2(x) + k_1 \right] e^{-x},\ \ C_2'(x) = \left[\alpha_2(x) + k_1 \right] e^{-x} .$$

Integrating these equalities from 0 to x, we find:

$$C_1(x) = -\int_0^x te^{-t}\alpha_2(t)dt + k_1\left(xe^{-x} + e^{-x} - 1 \right) + k_2 ,$$

$$C_2(x) = \int_0^x e^{-t}\alpha_2(t)dt + k_1\left(1 - e^{-x} \right) + k_3 .$$

Substituting these values in (37), we obtain

$$\tau_1(x)=\int_0^x (x-t)e^{x-t}\alpha_2(t)dt+k_1\left(xe^x-e^x+1\right)+\left(k_2+k_3x\right)e^x . \quad (40)$$

Differentiating (40) twice consecutively, we find

$$\tau_1'(x)=\int_0^x e^{x-t}\alpha_2(t)dt+\int_0^x (x-t)e^{x-t}\alpha_2(t)dt+k_1xe^x+k_2e^x+k_3(x+1)e^x , \quad (41)$$

$$\tau_1''(x)=\alpha_2(x)+2\int_0^x e^{x-t}\alpha_2(t)dt+\int_0^x (x-t)e^{x-t}\alpha_2(t)dt+k_1(x+1)e^x+k_2e^x+k_3(x+2)e^x . \quad (42)$$

Now substituting (40), (41), and (442) into conditions (29), respectively, we find

$$k_2=\frac{2}{2+c}f_1(0),\ \ k_3=f_1'(0)-f_1(0),\ \ k_1=f_1''(0)-\alpha_2(0)-k_2-2k_3 .$$

Finally, consider case 3) ($c=0$). In this case, equation (2-8) takes the form

$$\tau_1''(x)-\tau_1'(x)=\alpha_2(x)+k_1 . \quad (43)$$

The characteristic equation of this equation has two distinct real roots $\lambda_1=0$, $\lambda_2=1$. Integrating (43) from 0 to x, we obtain

$$\tau_1'(x)-\tau_1(x)=\alpha_3(x)+k_1x+k_2 , \quad (44)$$

where $\alpha_3(x)=\int_0^x \alpha_2(t)dt$, k_2 is an unknown constant.

The general solution of equation (44) has the form

$$\tau_1(x)=\int_0^x e^{x-t}\alpha_3(t)dt+k_1\left(e^x-1-x\right)+k_2\left(e^x-1\right)+k_3e^x . \quad (45)$$

Differentiating (45) twice consecutively, we find

$$\tau_1'(x)=\alpha_3(x)+\int_0^x e^{x-t}\alpha_3(t)dt+k_1\left(e^x-1\right)+k_2e^x+k_3e^x , \quad (46)$$

$$\tau_1''(x)=\alpha_2(x)+\alpha_3(x)+\int_0^x e^{x-t}\alpha_3(t)dt+k_1e^x+k_2e^x+k_3e^x . \quad (47)$$

Now substituting (45), (46) and (47) into conditions (29), respectively, we find

$$k_2=f_1(0),\ \ k_3=f_1'(0)-f_1(0),\ \ k_1=f_1''(0)-f_1'(0)-\alpha_2(0).$$

Thus, the function $\tau_1(x)$ is found, and therefore the functions– $\nu_1(x), \mu_1(x),\ u_{21}(x,y)$ are defined, and thus the function $u_2(x,y)$ is defined.

Now go to the area G_3. Passing in equation (19) ($i=3$) to the limit at $y\to 0$ and in the resulting equation changing x to $x-y$, we find

$$\omega_{32}(x-y)=f_1''(x-y)-f_3(x-y),\ -1\le x-y\le 0 .$$

Now consider the following auxiliary task:

$$\begin{cases} u_{3xx} - u_{3yy} = \Omega_3(x-y)e^{-cy}, \\ u_3(x,0) = F_1(x),\ u_{3y}(x,0) = F_2(x),\ -2 \le x \le 1, \\ u_3(-1,y) = \varphi_2(y),\ u_{3x}(-1,y) = \varphi_3(y),\ u_3(0,y) = \tau_2(y),\ 0 \le y \le 1. \end{cases} \quad (48)$$

We will look for a solution to this problem that satisfies all conditions except the condition $u_{3x}(-1,y) = \varphi_3(y)$, will be in the form

$$u_3(x,y) = u_{31}(x,y) + u_{32}(x,y) + u_{33}(x,y), \quad (49)$$

where $u_{31}(x,y) -$ is the solution to the problem

$$\begin{cases} u_{31xx} - u_{31yy} = 0, \\ u_{31}(x,0) = F_1(x),\ u_{31y}(x,0) = 0,\ -2 \le x \le 1, \\ u_{31}(-1,y) = \varphi_2(y),\ u_{31}(0,y) = \tau_2(y),\ 0 \le y \le 1 \end{cases} \quad (50)$$

$u_{32}(x,y) -$ peproblem solving

$$\begin{cases} u_{32xx} - u_{32yy} = 0, \\ u_{32}(x,0) = 0,\ u_{32y}(x,0) = F_2(x),\ -2 \le x \le 1, \\ u_{32}(-1,y) = 0,\ u_{32}(0,y) = 0,\ 0 \le y \le 1 \end{cases} \quad (51)$$

$u_{33}(x,y) -$ problem solving

$$\begin{cases} u_{33xx} - u_{33yy} = \Omega_3(x-y)e^{-cy}, \\ u_{33}(x,0) = 0,\ u_{33y}(x,0) = 0,\ -2 \le x \le 1, \\ u_{33}(-1,y) = 0,\ u_{33}(0,y) = 0,\ 0 \le y \le 1. \end{cases} \quad (52)$$

Here, the functions $F_1(x)$, $F_2(x)$ and $\Omega_3(x-y)$ are defined as follows: in the interval $-1 \le x \le 0$, the functions $F_1(x)$and $F_2(x)$ are known: $F_1(x) = f_1(x), F_2(x) = f_2(x)$, and in the intervals $-2 \le x \le -1$and $0 \le x \le 1$oare not yet known; and the function $\Omega_3(x-y)$ is defined asfollows: in the interval$-1 \le x-y \le 0$, it is known:$\Omega_3(x-y) = \omega_3(x-y)$, and in the intervals $-2 \le x-y \le -1$ and $0 \le x-y \le 1$ ona is still unknown.

The solution of problem (50)satisfying the first two conditions has the form

$$u_{31}(x,y) = \frac{1}{2}\left[F_1(x+y) + F_1(x-y)\right]. \quad (53)$$

Substituting (53) into the third condition of problem (50), we find

$$F_1(-1-y) = 2\varphi_2(y) - f_1(y-1),\ 0 \le y \le 1.$$

Here m $-1-y$ y na x, we get

$$F_1(x) = 2\varphi_2(-1-x) - f_1(-2-x),\ -2 \le x \le -1.$$

Substituting (53) into the fourth condition of problem (50), we find

$$F_1(y) = 2\tau_2(y) - f_1(-y),\ 0 \le y \le 1.$$

So, we have

$$F_1(x)=\begin{cases}2\varphi_2(-1-x)-f_1(-2-x),\ -2\le x\le -1\\ f_1(x),\ -1\le x\le 0,\\ 2\tau_2(x)-f_1(-x),\ 0\le x\le 1.\end{cases}$$

The solution of problem (51)satisfying the first two conditions has the form

$$u_{32}(x,y)=\frac{1}{2}\int_{x-y}^{x+y}F_2(t)dt\ . \tag{54}$$

Substituting (54) into the third condition of problem (51), we find

$$F_2(-1-y)=-f_2(y-1),\ 0\le y\le 1.$$

Here, changing $-1-y$ and x, we get

$$F_2(x)=-f_2(-2-x),\ -2\le x\le -1.$$

Now substituting (54) into the fourth condition of problem (51), we have

$$F_2(y)=-f_2(-y),\ 0\le y\le 1.$$

Means

$$F_2(x)=\begin{cases}-f_2(-2-x),\ -2\le x\le -1\\ f_2(x),\ -1\le x\le 0,\\ -f_2(-x),\ 0\le x\le 1.\end{cases}$$

The solution of problem (52)that satisfies the first two conditions has the form

$$u_{33}(x,y)=-\frac{1}{2}\int_0^y e^{-c\eta}d\eta\int_{x-y+\eta}^{x+y-\eta}\Omega_3(\xi-\eta)d\xi\ . \tag{55}$$

Substituting (55) into the third condition of problem (52), we obtain

$$\int_0^y e^{-c\eta}\Omega_3(y-1-2\eta)d\eta=-\Omega_3(-1-y)\int_0^y e^{-c\eta}d\eta\ . \tag{56}$$

Now substituting (55) into the fourth condition of problem (52), we have

$$\int_0^y e^{-c\eta}\Omega_3(y-2\eta)d\eta=-\omega_{32}(-y)\int_0^y e^{-c\eta}d\eta\ . \tag{57}$$

If we replace the integral on the left side of equality (57), we make a replacement $y-2\eta=z$, then it takes the form

$$\int_{-y}^{y} e^{-\frac{c}{2}(y-z)}\Omega_3(z)dz=-2\omega_{32}(-y)\int_0^y e^{-c\eta}d\eta\ .$$

Differentiation deriving this equality and taking into account the same equality, we find

$$\Omega_3(y)=[2\omega_{32}'(-y)-c\omega_{32}(-y)]\int_0^y e^{-c\eta}d\eta-3e^{-cy}\omega_{32}(-y)$$

Now substituting (53), (54) and (55) into (49), we obtain

$$u_3(x,y)=\frac{1}{2}\left[F_1(x+y)+F_1(x-y)\right]+\frac{1}{2}\int\limits_{x-y}^{x+y}F_2(t)dt-\frac{1}{2}\int\limits_0^y e^{-c\eta}d\eta\int\limits_{x-y+\eta}^{x+y-\eta}\Omega_3(\xi-\eta)d\xi\,. \quad (58)$$

Differentiation (58) by x, we find

$$u_{3x}(x,y)=\frac{1}{2}\left[F_1'(x+y)+F_1'(x-y)\right]+\frac{1}{2}\left[F_2(x+y)-F_2(x-y)\right]-$$

$$-\frac{1}{2}\int\limits_0^y e^{-c\eta}\left[\Omega_3(x+y-2\eta)-\Omega_3(x-y)\right]d\eta\,. \quad (59)$$

Assuming in (59) $x=-1$and taking into account the condition $u_{3x}(-1,y)=\varphi_3(y)$ after some calculations, we find

$$\Omega_3(-1-y)=\frac{c}{2}\left[f_1'(y-1)+f_2(y-1)-\varphi_2'(y)-\varphi_3(y)\right]e^{cy}-e^{cy}\omega_{32}(y-1)+$$

$$+2e^{cy}\left[f_1''(y-1)+f_2'(y-1)-\varphi_2''(y)-\varphi_3'(y)\right]$$

Assuming in (59) $x=0$ and taking into account the condition $u_{3x}(0,y)=\nu_2(y)$ after some transformations, we obtain

$$\nu_2(y)=\tau_2'(y)+\beta_1(y),\ 0\le y\le 1, \quad (60)$$

where

$$\beta_1(y)=f_1'(-y)-f_2(-y)+\omega_{32}(-y)\int\limits_0^y e^{-c\eta}d\eta\,.$$

Finally, go to the area G_1. Passing in equations (18) and (19) ($i=3$) to the limit at $x\to 0$, we obtain the relations

$$\mu_2(y)-\tau_2'(y)=\omega_{11}(-y)e^{-cy},\quad \mu_2(y)-\tau_2''(y)=\omega_{32}(-y)e^{-cy},$$

where the notation is entered $\omega_1(x-y)=\begin{cases}\omega_{11}(x-y), & -1\le x-y\le 0,\\ \omega_{12}(x-y), & 0\le x-y\le 1.\end{cases}$

Andtaking a function from these relations $\mu_2(y)$, we find

$$\omega_{11}(-y)=\omega_{32}(-y)+\left[\tau_2''(y)-\tau_2'(y)\right]e^{cy}\,.$$

Here, changing $-y$ na $x-y$, we get

$$\omega_{11}(x-y)=\omega_{32}(x-y)+\left[\tau_2''(y-x)-\tau_2'(y-x)\right]e^{c(y-x)}\,. \quad (61)$$

Passing in equation (18) to the limit at $y\to 0$, we find

$$\omega_{12}(x)=\tau_1''(x)-\nu_1(x),\ 0\le x\le 1\,.$$

Now we write down the solution of equation (18)that satisfies the conditions (2), (11), (14):

$$u_1(x,y)=\int\limits_0^y\tau_2(\eta)G_\xi(x,y;0,\eta)d\eta-\int\limits_0^y\varphi_1(\eta)G_\xi(x,y;1,\eta)d\eta+\int\limits_0^1\tau_1(\xi)G(x,y;\xi,0)d\xi-$$

$$-\int_0^y e^{-c\eta}d\eta\int_0^\eta \omega_{11}(\xi-\eta)G(x,y;\xi,\eta)d\xi-\int_0^y e^{-c\eta}d\eta\int_\eta^1 \omega_{12}(\xi-\eta)G(x,y;\xi,\eta)d\xi. \qquad (62)$$

Differentiating (62) by x, after some calculations, we obtain

$$u_{1x}(x,y)=-\int_0^y \tau_2'(\eta)N(x,y;0,\eta)d\eta+\int_0^y \varphi_1'(\eta)N(x,y;1,\eta)d\eta+\int_0^1 \tau_1'(\xi)N(x,y;\xi,0)d\xi+$$

$$+\int_0^y e^{-c\eta}d\eta\int_0^\eta \omega_{11}(\xi-\eta)N_\xi(x,y;\xi,\eta)d\xi+\int_0^y e^{-c\eta}d\eta\int_\eta^1 \omega_{12}(\xi-\eta)N_\xi(x,y;\xi,\eta)d\xi. \qquad (63)$$

Assuming in (63) $x\to 0$, we have

$$\nu_2(y)=-\int_0^y \tau_2'(\eta)N(0,y;0,\eta)d\eta+\int_0^y \varphi_1'(\eta)N(0\,y;1,\eta)d\eta+\int_0^1 \tau_1'(\xi)N(0,y;\xi,0)d\xi+$$

$$+\int_0^y e^{-c\eta}d\eta\int_0^\eta \omega_{11}(\xi-\eta)N_\xi(0,y;\xi,\eta)d\xi+\int_0^y e^{-c\eta}d\eta\int_\eta^1 \omega_{12}(\xi-\eta)N_\xi(0\,y;\xi,\eta)d\xi. \qquad (64)$$

Substituting (60) and (61) in (64) ga kyyib, after long calculations and transformations, we arrive at the Volterr integral equation of the second kind of relative $\tau_2''(y)$:

$$\tau_2(y)+\int_0^y K(y,\eta)\tau_2''(\eta)d\eta=g(y),$$

where $K(y,\eta)$ and $g(y)$ are known functions, and $K(y,\eta)$ has a weak singularity (with degree $1/2$), and $g(y)$ is continuous. Therefore, this equation admits a unique solution in the class of continuous functions. Solving it, we find $\tau_2''(y)$, and thus define the functions $\tau_2(y)$, $\nu_2(y)$, $\omega_{11}(x-y)$, $u_3(x,y)$, $u_1(x,y)$.

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