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ABOUT THE CONTINUITY OF THE FUNCTION, THE SMOOTH CONTINUITY, AND THE CONNECTIONS BETWEEN THE DERIVATIVE

B.Mamadaliyev
Candidate of physical and Mathematical Sciences of the Kokand State Pedagogical Institute

N.Z.Mamadaliyeva
Associate professor of Kokand State Pedagogical Institute

Annotation. This article describes analyses of the continuity of a function, the smooth continuity, and the connections between the derivative.

Keywords: continuity of function, smooth continuity, derivative, theorem.

As we know, the concepts of continuity, smooth continuity and derivative of a function are among the basic concepts of mathematical analysis. These concepts have different meanings in relation to each other, and the relationship between them also has different character. That is, it follows that a continuous function defined on a set is always continuous on that set, while it does not always follow that the function is smooth continuous, it follows that the function is always continuous on a plane continuous. From the fact that a function is continuous on some set to every point of its same set, while the existence of a derivative does not always follow, all the time from the existence of derivatives of a function on some set comes its continuity on that set.

There is information about this relationship textbook, the teaching is given enough in the literature.

It is also possible to say the same reasoning about the relationship between the smooth continuum and derivative of a function on a set, although there is not enough information about it in the available literature.

Such relations depend on the conditions under which a function is defined on an open or closed set and is put on its derivative.

As you know from the course of mathematical analysis, $[a,b]$ a function defined in a closed range is continuous if it is flat continuous in that range. ([1], Contor's theorem). Also, if the function $[a,b]$ if there are derivatives in the closed range, then the function is continuous and Planar continuous in that range. This affirmation also follows directly from the relationship between the existence and continuity of the derivative of the function and from Contor's theorem. It is possible to obtain information available literature.

But (a,b), $(a,b]$, $[a,b)$ for functions when defined in the open range of view, such relations are not sufficiently covered in the existing teaching literature.

Our goal is to learn more broadly in the relationship between continuity, smooth continuity, and derivative of a function for functions defined over open intervals.

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About the authors：B.Mamadaliyev
Corresponding author- Email:

X the set was an arbitrary open set.

Theorem. If $f(x)$ of the function X has a derivative on the set and is bounded, then the function X the set is flat continuous.

Proof. $f(x)$ of the function X let the derivative on the set exist and be bounded. $f(x)$ of the function X we show that the set is flat continuous.

According to the theorem representing the connection between the smooth continuity of a function and its continuity modulus [1]

$$\lim_{s\to+0}\omega(s)=\lim_{s\to+0}\sup_{|x_1-x_2|<s}\omega(s)\left\{\left|f(x_1)-f(x_2)\right|\right\}=0 \tag{1}$$

it is sufficient to show that the plane is appropriate. (here $x_1,x_2\in X$).

According to the [1] Logrange theorem in the fundamental theorems of differential calculus

$$\lim_{s\to+0}\sup_{|x_1-x_2|<s}\left\{\left|f(x_1)-f(x_2)\right|\right\}=$$

$$=\lim_{s\to+0}\sup_{|x_1-x_2|<s}\left\{\left|f`(\xi)\right|\cdot\left|x_1-x_2\right|\right\}\le$$

$$\le\lim_{s\to+0}\left|f`(\xi)\right|\cdot s \tag{2}$$

(here). $\xi\in[x_1,x_2]$ or $\xi\in[x_2,x_1]$).

According to the theorem condition $\left|f`(\xi)\right|<C$ (C-invariant) since is bounded from (1) and (2) planes

$$\lim_{s\to+0}\omega(s)=0$$

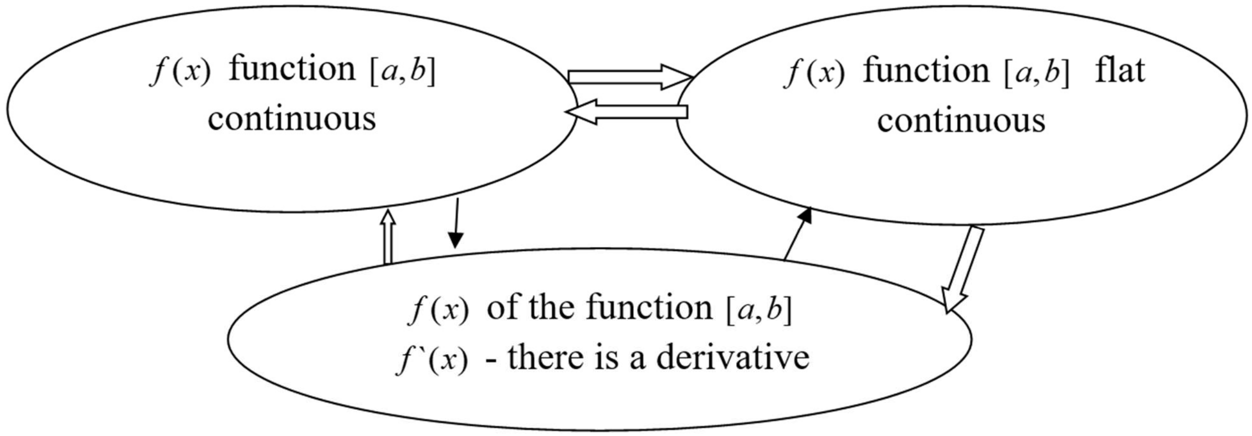
attitude is derived from relevance.

Hence, according to the theorem about the continuity modulus and the smooth continuous [1] $f(x)$ function X the set is flat continuous.

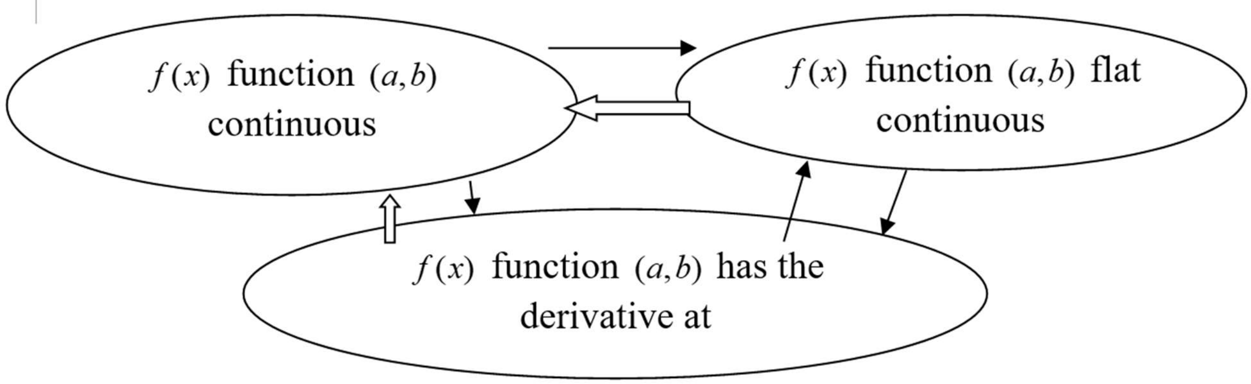
The theorem proved.

It follows from the confirmation of the theorem and from the above considerations that it becomes possible to describe the relationship between the continuity of the function, the smooth continuity and the derivative in schematic terms as follows:

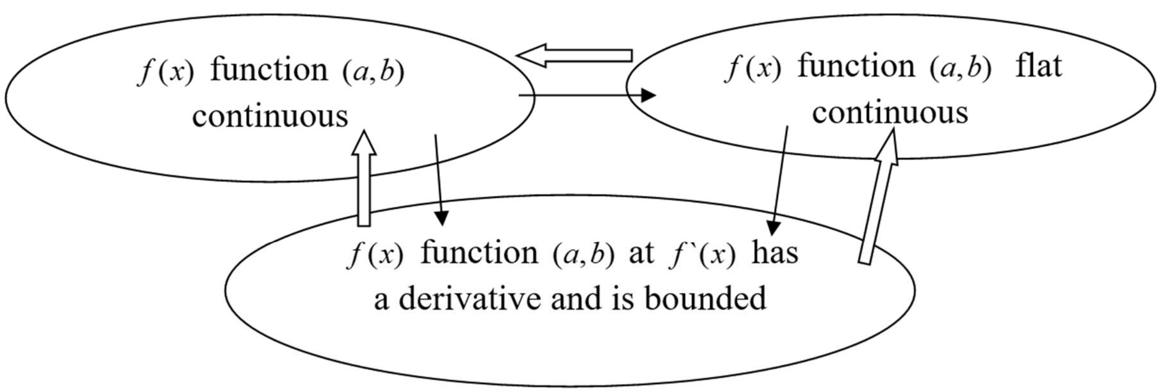
a) $f(x)$ function $[a,b]$ let be defined in the closed range.



b) $f(x)$ function (a,b) let be defined in the open range.



s) $f(x)$ function (a,b) let be defined in the open range.



Note: $\Longrightarrow$- the sign comes all the time, $\rightarrow$ and the sign means that in some cases it comes, in some cases it does not come.

b) and s) cases $f(x)$ function $[a,b)$ and $(a,b]$ it will also be appropriate for cases identified at intervals.

Based on the importance of the concepts of continuous, flat continuity and derivative of a function in mathematical analysis and other disciplines, we think that the coverage of such bindings and the above-mentioned theorem in mathematical analysis and textbook on higher mathematics, educational literature positively affects students to understand topics more broadly and perfectly.

References

1. T.Azlarov, H.Mansurov. “Matematik analiz”. 1-qism. Toshkent, O‘qituvchi, 1994 yil.
2. G.M.Fixtengols. “Matematik analiz asoslari”. 1-qism. Toshkent, O‘qituvchi, 1972 yil.
3. A.Sa’dullayev va boshqalar. “Matematik analiz kursidan misol va masalalar to‘plami”. Toshkent, O‘qituvchi, 1993 yil.