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TO ANALYSIS THE VIBRATION OF NON-HOMOGENEOUS PARALLELOGRAM SKEW PLATE WITH CIRCULAR VARIATION THICKNESS WITH THERMAL EFFECT

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Abstract

The structure of high speed air-craft i.e. supersonic flights, missiles are subjected to high surface temperature and large thermal gradient. These conditions can damage the structure of vehicles. Variation in structures' characteristics as a result of thermal effect can be observed by changes in frequency of vibration. In present paper we investigate the free vibration of isotropic parallelogram plate under the effect of bi-linear thickness and bi-linear temperature distribution in both directions. The frequency values for the primary two modes of vibration have been calculated for a simply supported parallelogram plate for various values of aspect ratio, , skew angle , thermal gradient and taper constants with the help of MAPLE (today's computational software). The Rayleigh-Ritz technique is used to calculate the frequency equation by two term deflection function.

Keywords: vibration**,** skew plate, thickness, taper constant, thermal gradient, non-homogeneity constant.

1. Introduction

Vibration is the mechanical oscillations of an item about an equilibrium factor. The oscillations may be normal which includes the motion of a pendulum or random including the movement of a tire on a gravel street. Vibration of Plates offers a comprehensive, self-contained advent to vibration theory and evaluation of two-dimensional plates. Vibrations are encountered in many mechanical and structural applications, for example, mechanisms and machines, homes, bridges, motors, and plane. The effects of thermally induced vibrations in large machines have always been a most important concern in the field of science and technology. It is desirable to design such large machines for smooth operation with organized vibrations. This analysis is also beneficial for civil and architectural engineers to build earthquake resistant constructions. Also, the tapered plates i.e. the plates with varying thickness variations are frequently used in many engineering and industrial applications.

In recent years, owing to the diversification of engineering materials and for operations in several thermal environments i.e. nuclear weapons, missiles, defense weapons, laser weapons etc. thermal problems have become very important for modern designers and researchers. The classical theory of vibration (which deals with the effect of thermal gradient on vibration) has attracted the attention of many researchers because of its extensive use in diverse fields. The plates of variable thickness are frequently used as structural components and their vibration characteristics are important for practical

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design. The structural components are used in many applications involving aerospace, submarine structural, civil engineering structures etc. Depending upon the requirement, durability and reliability materials are being developed so that these may provide better strength, efficiency and economy. Therefore a study of character and

Behavior of these plates is required so that the full potential of these plates may be used. These plates may be of any type for example-rectangular plate, square plate, triangular plate. Also, the thickness of these plates affects the behavior of plates.

In last few years, a lot of research has been performed in the field of vibration of plate structures of various shapes and sizes. An up-date literature survey is as follows: An extensive review on linear vibration of plates has been given by Leissa [11] in his monograph. The Ritz method is employed for the all the results. Conwey and Farnham [6] study the free vibration of triangular, rhombic and parallelogram plates. The different skew angles of simply supported and clamped boundary conditions for frequencies were calculated. Leissa [27], [31] presented plate vibration research, classical theory and Plate vibration research, complicating effects. Jain and Soni [14] analyzed Free Vibration of rectangular plates of parabolic ally varying thickness .Gupta and Khanna [131] analyzed vibration of a visco-elastic rectangular plate under the effect of linearly varying thickness in both directions. Sharma Amit [150]. The present study analyzes the natural vibration of non homogeneous visco elastic skew plate (parallelogram plate) with non uniform thickness under temperature field. Here non homogeneity in the plate's material arises due to circular variation in Poisson's ratio. Gupta, Kumar [143] study the vibration of visco-elastic parallelogram plate whose thickness varies parabolic ally. It is assumed that the plate is clamped on all the four edges and that the thickness varies parabolic ally in one direction i.e. along length of the plate.

2. Analysis

The parallelogram (skew) plate is assumed to be non-uniform, thin and isotropic and the plate R be defined by the three number a, b and θ.

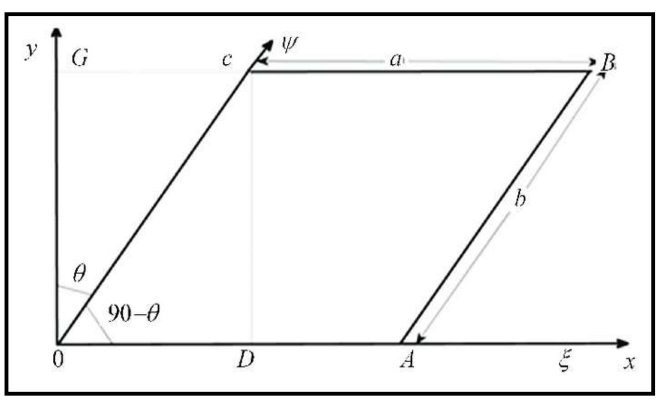


Figure 2.1. The parallelogram plate with skew angle θ

The skew coordinates of the plate are:

$$x = \xi + \varphi \sin\theta , \; y = \varphi \cos\theta \quad (1)$$

Using eqn. (1), the equation of K.E. and Strain energy will become:

$$M_E = \frac{1}{2} k^2 \rho \cos\theta \int_0^b \int_0^a l W^2 \, d\xi d\varphi \quad (2)$$

$$N_E = \frac{1}{2}\int_0^b \int_0^a D_1 [(W_{,\xi\xi})^2 - 4\sin\theta (W_{,\xi\xi})(W_{,\xi\varphi}) + 2 (\sin^2\theta + \nu\cos^2\theta)(W_{,\xi\xi})(W_{,\varphi\varphi}) + 2 (1 + \sin^2\theta - \nu\cos^2\theta) (W_{,\xi\varphi})^2 - 4\sin\theta(W_{,\xi\varphi})(W_{,\varphi\varphi}) + (W_{,\varphi\varphi})^2] d\xi d\varphi$$

(3)

3. Assumptions

1. The thickness of the plate is assumed to be circular in two dimensions.

$$\ell = \ell_0 \left[1 + \beta_1 \left(1- \sqrt{1-\frac{\xi}{a}}\right)\right]\left[1 + \beta_2 \left(1- \sqrt{1-\frac{\varphi}{b}}\right)\right] \quad (4)$$

Where β_1, β_2 is tapering constant. Thickness of the plate becomes constant at $\xi = 0 , \varphi = 0$.

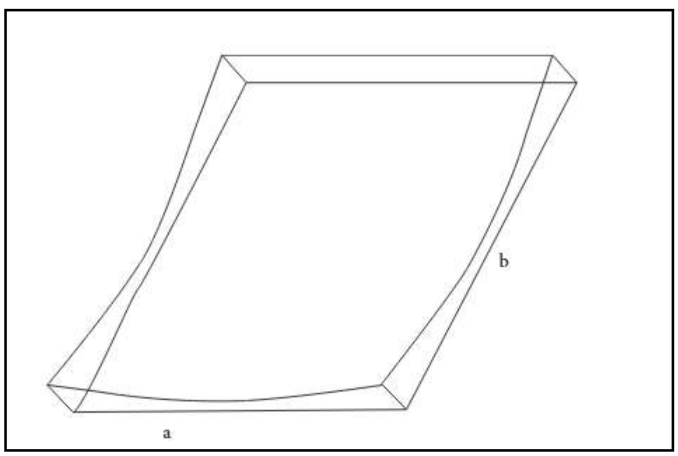


Fig.2.2 Parallelogram plate having two-dimensional circular thickness.

2. We consider plate's material to be non-homogeneous. Therefore, either density or Poisson's ratio varies circularly in two dimensions as :

$$\nu = \nu_0 \left([1 - m \left(1 - \sqrt{1-\frac{\xi}{a}} \right)]\right) \quad (5)$$

Where m_1, m_2 is known as non-homogeneity constant. Poisson's ratio becomes constant i.e $\nu = \nu_0$ at $\xi = 0$, $\varphi = 0$.

3. The temperature variation on the plate is considered to be bilinear i.e. linear in ξ direction and linear in φ direction as :

$$\eta = \eta_0 (1- \frac{\xi}{a}) (1- \frac{\varphi}{b}) \qquad (6)$$

Where η and η_0 denotes the temperature excess above the reference temperature on the plate at any point and at the origin .The temperature dependence modulus of elasticity for engineering structures is given by:

$$Y = Y_0 (1 - \gamma \eta) \qquad (7)$$

Where Y_0 is the Young's Modulus at mentioned temperature (i.e. $\eta = 0$) and γ is called slope of variation.

Using equation (6) in equation (7), we get:

$$Y = Y_0 [1 - \alpha(1- \frac{\xi}{a}) (1- \frac{\varphi}{b})] \qquad (8)$$

Where α, $(0 \le \alpha < 1)$ is called temperature, which is the product of temperature at origin and γ slope of variation i.e. gradient $\alpha = \gamma\eta_0$.

Using equation (4), (5) and (8), flexural rigidity i.e. $D_1 = \frac{yl^3}{(1-\nu)^2}$ of the plate becomes:

$$D_1 = \frac{Y0\ [1 - \alpha(1- \frac{\xi}{a})(1- \frac{\varphi}{b})]\ l0\ [1 + \beta1 (1- \sqrt{1-\frac{\xi}{a}})][1 + \beta2 \left(1- \sqrt{1-\frac{\varphi}{b}}\right)]^3}{12((1-\nu_0{}^2 [1 - m (1- \sqrt{1-\frac{\xi}{a}})]^2)} \qquad (9)$$

Using (4), (5) and (9), the eqn. of K.E. and Strain Energy becomes:

$$M_E = \frac{1}{2} k^2 \rho l_0 \int_0^b \int_0^a (1 + \beta_1 C1)(1 + \beta_2 C2)\ W^2 d\xi d\varphi \qquad (10)$$

$$N_E = \frac{y_0 l_0}{24 \cos^4\theta} \int_0^b \int_0^a \left[\frac{\left[1 - \alpha\left(1- \frac{\xi}{a}\right)\left(1- \frac{\varphi}{b}\right)\right][(1+\beta_1 C1)(1+\beta_2 C2)]^3}{(1-\nu_0{}^2 ([1 - m C1])^2}\right] [(W_{,\xi\xi})^2 - 4\left(\frac{a}{b}\right)\sin\theta\,(W_{,\xi\xi})(W_{,\xi\varphi}) + 2\left(\frac{a}{b}\right)(\sin^2\theta + \nu_0 t\,[1 - mC1]\cos^2\theta)t(W_{,\xi\xi})(W_{,\varphi\varphi}) + 2\left(\frac{a}{b}\right)^2 (1 + \sin^2\theta - \nu_0\,[1 - mC1]\cos^2\theta)\,(W_{,\xi\varphi})^2 - 4\left(\frac{a}{b}\right)^3 \sin\theta (W_{,\xi\varphi})(W_{,\varphi\varphi}) + \left(\frac{a}{b}\right)^4 (W_{,\varphi\varphi})^2]]d\xi d\varphi \qquad (11)$$

Where,

$$C1 = (1- \sqrt{1 - \frac{\xi}{a}}),\ C2 = (1- \sqrt{1 - \frac{\varphi}{b}})$$

In this paper, we are calculating first two mode of vibration on clamped boundary condition, therefore we have:

$$\left.\begin{aligned} W=W_{,\xi} = 0 \text{ at } \xi = 0, a \\ W=W_{,\varphi} = 0 \text{ at } \varphi = 0, b \end{aligned}\right\} \quad (12)$$

Hence, the two term deflection function, which satisfies eqn. (17), is:

$$W(\xi,\varphi) = [A1 \left(\frac{\xi}{a}\right)^2 \left(\frac{\varphi}{b}\right)^2 \left(1-\frac{\xi}{a}\right)^2 \left(1-\frac{\varphi}{b}\right)^2 + A_2 \left(\frac{\xi}{a}\right)^3 \left(\frac{\varphi}{b}\right)^3 \left(1-\frac{\xi}{a}\right)^3 \left(1-\frac{\varphi}{b}\right)^3]$$

$$W(\xi,\varphi) = \left(\frac{\xi}{a}\right)^2 \left(\frac{\varphi}{b}\right)^2 \left(1-\frac{\xi}{a}\right)^2 \left(1-\frac{\varphi}{b}\right)^2 [A1 + A_2 (\frac{\xi}{a})(\frac{\varphi}{b}) (1-\frac{\xi}{a}) \left(1-\frac{\varphi}{b}\right)] \quad (13)$$

Where A_1 and A_2 are arbitrary constant.

4. Solution for frequency equation by Rayleigh-Ritz method

We used Rayleigh-Ritz method to solve frequency equation and frequency mode i.e. in Rayleigh-Ritz method maximum kinetic energy must be equal to maximum strain energy.

Hence we have:

$$\delta (N_E - M_E) = 0 \quad (14)$$

Using equation (10) and (11) , we get :

$$\delta (N_E^* - \lambda^2 M_E^*) = 0 \quad (15)$$

Where,

$$M_E^* = \int_0^b \int_0^a (1+\beta_1 C1)(1+\beta_2 C2)\; W^2 d\xi d\varphi \quad (16)$$

And

$$N_E^* = \frac{1}{cos^4\theta} \int_0^b \int_0^a \{\left[\frac{\left[1-\alpha\left(1-\frac{\xi}{a}\right)\left(1-\frac{\varphi}{b}\right)\right][(1+\beta_1 C1)(1+\beta_2 C2)]^3}{(1-\nu_0{}^2 ([1 - mC]^2}\right] [(W_{,\xi\xi})^2 - 4\left(\frac{a}{b}\right)\sin\theta (W_{,\xi\xi})(W_{,\xi\varphi}) + 2\left(\frac{a}{b}\right)(\sin^2\theta + \nu_0 t [1 - mC1]\cos^2\theta) t (W_{,\xi\xi})(W_{,\varphi\varphi}) + 2\left(\frac{a}{b}\right)^2 (1+\sin^2\theta - \nu_0 [1 - mC1]\cos^2\theta)(W_{,\xi\varphi})^2 - 4\left(\frac{a}{b}\right)^3 \sin\theta (W_{,\xi\varphi})(W_{,\varphi\varphi}) + \left(\frac{a}{b}\right)^4 (W_{,\varphi\varphi})^2]]\} d\xi d\varphi \quad (17)$$

And $\lambda^2 = \frac{12\omega^2 a^4 \rho}{Y0\ell_0{}^2}$ is known as frequency parameter.

Equation (14) consists of 2 unknown constants (A_1 &A_2) which are obtained by the substitution of W and these constant can be evaluated by the following formula:

$$\frac{\partial}{\partial A_1}(N_E^* - \lambda^2 M_E^*) = 0, \quad \frac{\partial}{\partial A_2}(N_E^* - \lambda^2 M_E^*) = 0 \quad (18)$$

after solving equation (18) , we get,

$$d_{11}A_1 + d_{12}A_2 = 0 \quad (19)$$

$$d_{21}A_1 + d_{22}A_2 = 0 \quad (20)$$

Where d_{11}, $d_{12} = d_{21}$ and d_{22} involve parametric constant and frequency parameter.

For a non- trivial solution the determinant of the coefficients of Equation (25) & (26) must be zero.

Therefore, we get the frequency equation,

$$\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = 0 \quad (21)$$

With the help of equation (21), we get quadratic equation in λ^2. We can obtain two roots of λ^2 from this equation. These roots give the first (λ_1) and second (λ_2) modes of vibration of frequency for various parameters.

5. Results and Discussion

The frequency (λ) for first and second mode of vibration of an isotropic (clamped) parallelogram plate has been determined for different values of thermal constant(α), tapering constant (β_1 and β_2), aspect ratio (a/b) and non-homogeneity constant (m) and skew angle(θ).Every one of the outcomes are acquired by utilizing MATLAB/MAPLE programming. All the results are shown with the help of Figures.

Following boundaries are utilized for this estimation are: v_0=0.345, a/b=1.5.

In Fig I: Thickness (tapering parameter (β_1) variation in plate v/s frequency (λ) with fixed value of θ = 30^0 and a/b = 1.5 and different values of taper constants and non-homogeneity constant ($\beta_1 = \beta_2 = m = \alpha$ = 0, 0.4, 0.8).From fig.1 that as value of taper constant (β_1) increases from 0 to 0.8 corresponding frequency value (λ) for 1st and 2nd mode of vibration also increases.

In Fig II: Thickness (tapering parameter (β_2) variation in plate v/s frequency (λ) for θ = 30^0 and a/b = 1.5 and different values of taper constants and non-homogeneity constant ($\beta_1 = \beta_2 = m = \alpha$ = 0, 0.4, 0.8).From fig.II that as value of taper constant (β_2) increases from 0 to 0.8 corresponding frequency value (λ) for 1st and 2nd mode of vibration increases.

In Fig III: non-homogeneity (m_1) variation in plates material v/s vibrational frequency (λ) for θ = 30^0 and a/b = 1.5 and different values of taper constants and non-homogeneity constant ($\beta_1 = \beta_2 = m = \alpha$ = 0, 0.4, 0.8).From fig.III that as value of non-homogeneity (m) increases from 0 to 0.8 corresponding frequency value (λ) for 1st and 2nd mode of vibration is decreases.

In Fig IV: Thermal gradient (α) variation in plates material v/s frequency (λ) for θ = 30^0 and a/b = 1.5 and different values of taper constants and non-homogeneity constant ($\beta_1 = \beta_2 = m$ = 0, 0.4, 0.8).From fig.IV that frequency mode decreases as value of thermal gradient increases from 0 to 0.8 i.e. Corresponding frequency value (λ) for 1st and 2nd mode of vibration decreases.

In Fig V: skew angle (θ) variation in plates material v/s frequency (λ) for a/b = 1.5 and different values of taper constants and non-homogeneity constant ($\beta_1 = \beta_2 = \alpha = 0.4$, m = 0, 0.4, 0.8).From fig.V that frequency mode increases sharply as value of skew angle increases from 0 to 75 i.e. Corresponding frequency value (λ) for 1st and 2nd mode of vibration increases.

Figure -1 Taper Constant (β_1) v/s Frequency (λ)

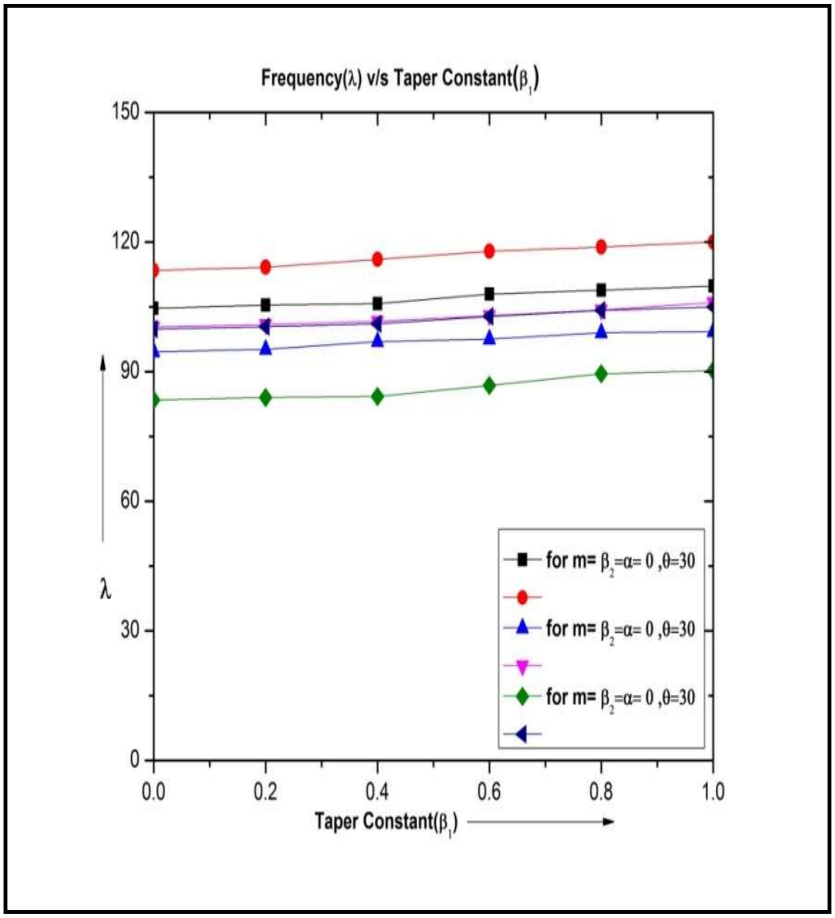


Figure -2 Taper Constant (β_2) v/s Frequency (λ)

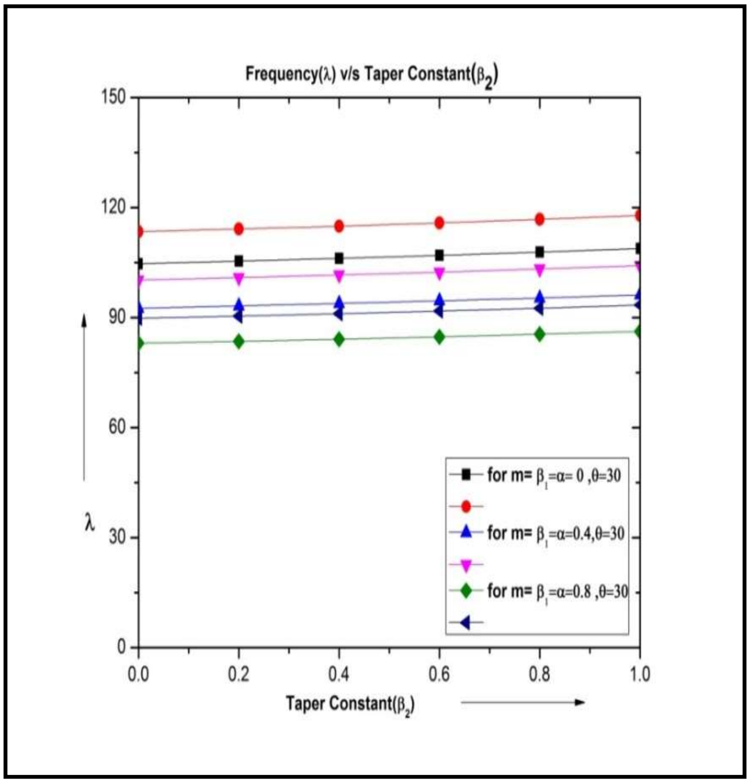


Figure -3 Non-Homogeneity (m) v/s Frequency (λ)

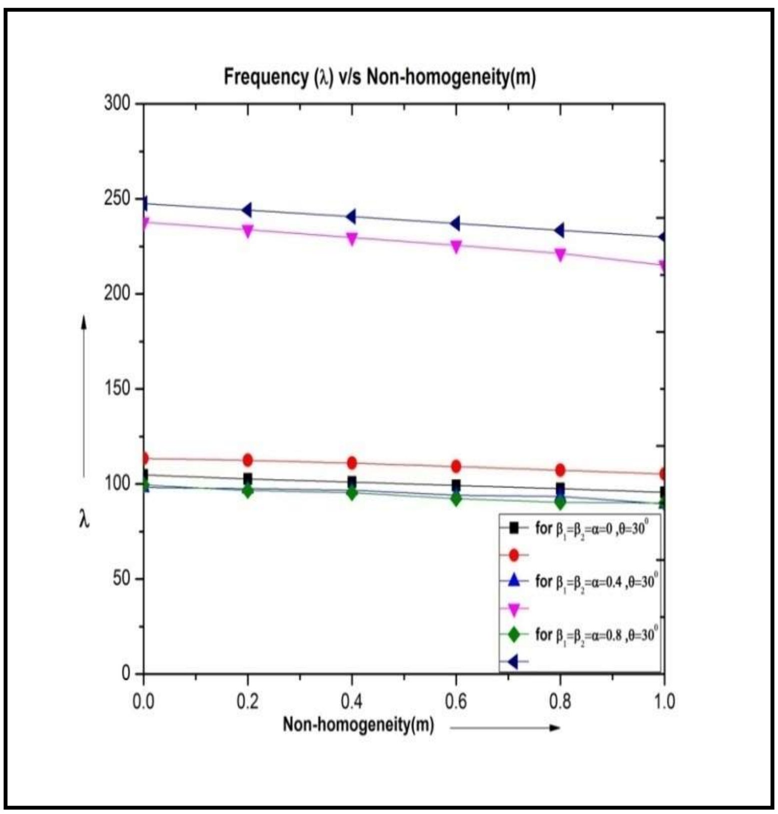


Figure -4 Thermal Gradients (α) v/s Frequency (λ)

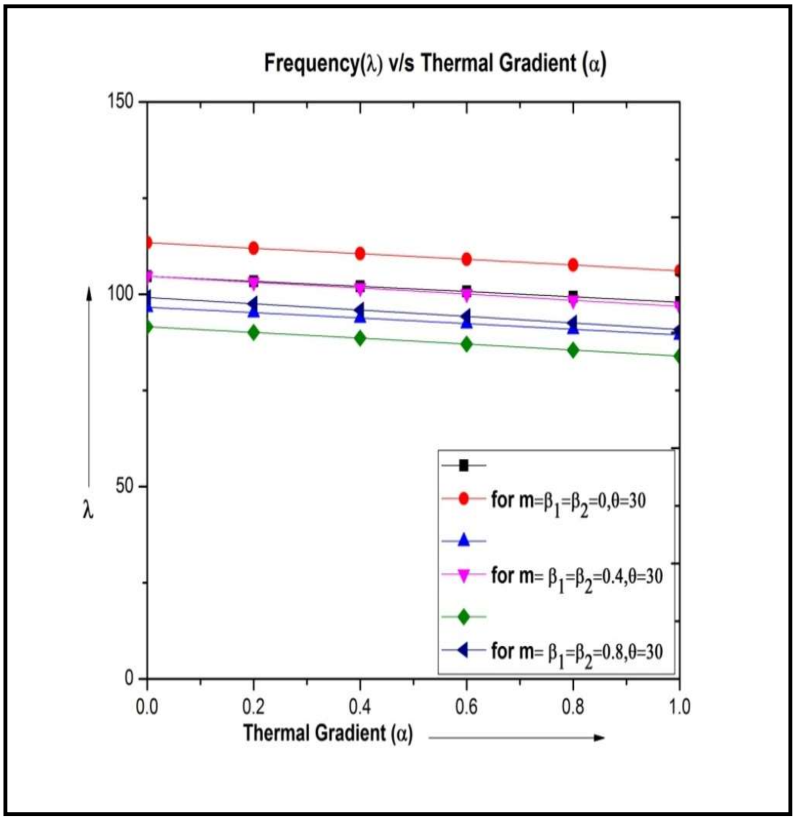
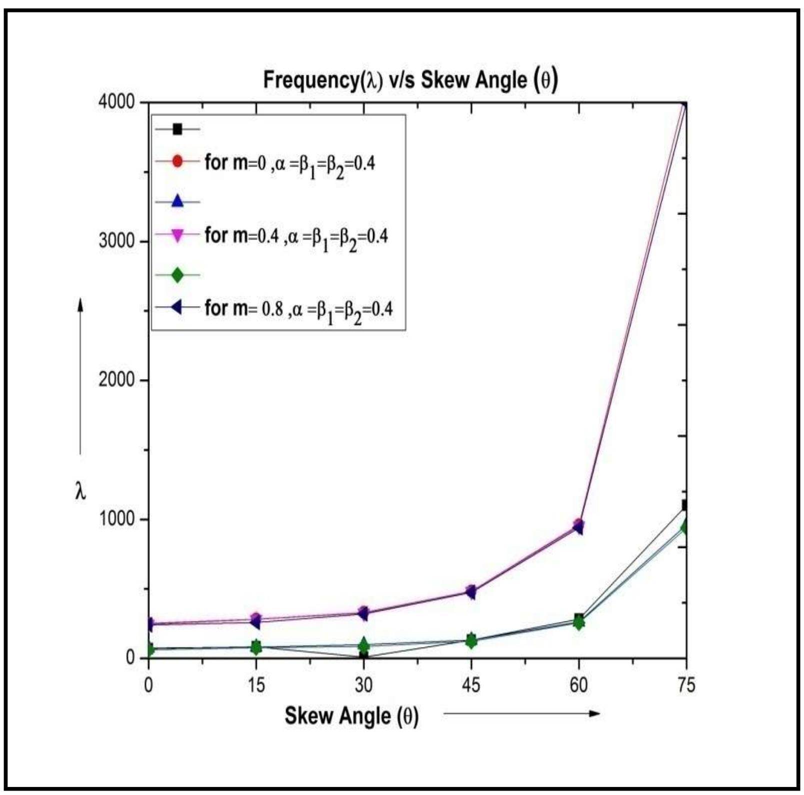


Figure -5 Skew Angle (θ) v/s Frequency (λ)



6. Conclusion

Rayleigh - Ritz technique is applied to study the effect of various parameters (taper constants, thermal constant, and non-homogeneity constant, skew angle) on vibration of non-homogeneous parallelogram skew plate with circular variation in bi-linear thickness and bi-linear temperature variation. From the result discussion author conclude that as tapering constant (β_1 and β_2) and skew angle (θ) increases, frequency increases for both modes of vibration. While it decreases as thermal gradient and Non-Homogeneity increases. This paper gives good appropriate numerical data of frequency modes which is helpful for researchers and scientists, making good optimal structural designs.

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